ABSTRACT:
In this paper generalized minimal closed maps that include a class of generalized minimal homeomorphisms and generalized minimal* homeomorphisms are introduced and studied in topological spaces. A bijective mapping $f: (X, \tau) \to (Y, \sigma)$, is said to be generalized minimal homeomorphism (briefly g- m$_i$ homeomorphism) if $f$ and $f^{-1}$ are g- m$_i$ continuous maps.

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1. INTRODUCTION AND PRELIMINARIES

Section 2 is a brief study of generalized minimal homeomorphisms and generalized minimal* homeomorphisms in topological spaces.

Throughout this paper $(X, \tau)$, $(Y, \sigma)$ and $(Z, \eta)$ denote topological spaces on which no separation axioms are assumed unless otherwise explicitly mentioned. For any subset $A$ of a topological space $(X, \tau)$, closure of $A$, interior of $A$ and complement of $A$ is denoted by $\text{cl}(A)$, $\text{int}(A)$ and $A^c$ respectively. We recall the following definitions, which are prerequisites for our present study.

**Definition 1.1:** A proper nonempty subset $A$ of a topological space $(X, \tau)$ is called

(i) a minimal open [5] (resp. minimal closed [7]) set if any open (resp. closed) subset of $X$ which is contained in $A$, is either $A$ or $\phi$.

(ii) a maximal open [6] (resp. maximal closed)[7]) set if any open (resp. closed) subset of $X$ which contains $A$, is either $A$ or $X$.

**Remark 1.2 [7]:** Minimal open (resp. minimal closed) sets and maximal closed (resp. maximal open) sets are complements of each other.

**Definition 1.3[2]:** A subset $A$ of a topological space $(X, \tau)$ is called

(i) a generalized closed[1] (briefly g-closed) set if $\text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is an open set in $X$.

(ii) a generalized open (briefly g-open ) set [1] iff $A^c$ is a g-closed set.

**Definition 1.4:** A mapping $f: (X, \tau) \rightarrow (Y, \sigma)$ is said to be

(i) generalized closed (g-closed) map [4] if the image of every closed set in $X$ is g-closed set in $Y$.

(ii) minimal open (resp. minimal closed) map if the image of every minimal open (resp. minimal closed) set in $X$ is an open (resp. closed) set in $Y$.

(iii) maximal open (resp.maximal closed) map if the image of every maximal open (resp. maximal closed) set in $X$ is an open (resp. closed) set in $Y$.

(iv) strongly minimal open (resp.strongly minimal closed) map if the image of every minimal open (resp. minimal closed) set in $X$ is minimal open (resp. minimal closed) set in $Y$.

(v) strongly maximal open (resp.strongly maximal closed) map if the image of every maximal open (resp. maximal closed) set in $X$ is maximal open (resp. maximal closed) set in $Y$.

(vi) minimal generalized open (resp. minimal generalized closed) map if the image of every minimal open (resp. minimal closed) set in $X$ is g-open (resp. g-closed) set in $Y$. 

(vii) maximal generalized open (resp. maximal generalized closed) map if the image of every maximal open (resp. maximal closed) set in X is g-open (resp. g-closed) set in Y.

**Definition 1.5:** A bijective mapping \( f: (X, \tau) \to (Y,\sigma) \) is said to be

(i) generalized homeomorphism [3] if \( f \) is g-continuous and g-open.

(ii) gc-homeomorphism [3] if \( f \) and \( f^{-1} \) are gc-irresolute maps.

(iii) minimal g-homeomorphism (resp. maximal g-homeomorphism) if \( f \) is minimal g-continuous (resp. maximal g-continuous) and minimal g-open (resp. maximal g-open).

**Definition 2.1:** A bijective mapping \( f: (X, \tau) \to (Y,\sigma) \), is said to be g-generalized minimal homeomorphism (briefly g-mi homeomorphism) if \( f \) and \( f^{-1} \) are g-minimal continuous maps.

**Theorem 2.2:** If \( f: (X, \tau) \to (Y,\sigma) \) is a bijective map, then the following statements are equivalent.

(i) Its inverse map \( f^{-1}: (Y, \sigma) \to (X, \tau) \) is g-mi continuous.

(ii) \( f \) is a g-mi open map.

(iii) \( f \) is a g-mi closed map.

**Proof:** (i) \( \Rightarrow \) (ii). Let \( V \) be any maximal open set in X, so that \( V^c \) is a minimal closed set in X. From (i) \( (f^{-1})^{-1}(V) = f(V^c) = (f(V))^c \) is a g-minimal closed set in Y, so that \( f(V) \) is a g-maximal open set in Y. Therefore \( f \) is a g-mi open map.

(ii) \( \Rightarrow \) (iii). Let \( U \) be any minimal closed set in X, so that \( U^c \) is a maximal open set in X. From (ii) \( f(U^c) = (f(U))^c \) is a g-maximal open set in Y, so that \( f(U) \) is a g-minimal closed set in Y. Therefore \( f \) is a g-mi closed map.

(iii) \( \Rightarrow \) (i). Let \( U \) be any minimal closed set in X. From (iii) \( f(U) \) is a g-minimal closed set in Y, so that \( (f^{-1})^{-1}(U) = (f(U))^c \) is a g-minimal closed set in Y. Therefore the inverse map \( f^{-1}: (Y, \sigma) \to (X, \tau) \) is g-mi continuous map.

**Theorem 2.3:** If \( f: (X, \tau) \to (Y,\sigma) \) is a bijective map and g-minimal continuous map, then the following statements are equivalent.

(i) \( f \) is a g-mi open map.

(ii) \( f \) is a g- mi homeomorphism.

(iii) \( f \) is a g-mi closed map.
Proof: (i) ⇒ (ii). Let U be a minimal closed set in X, so that U^c is a maximal open set in X. From (i) \( f(U^c) = (f(U))^c \) is a g-maximal open set in Y, so that \( f(U) = (f^{-1})^c(U) \) is g-minimal closed set in Y. Therefore \( f^{-1}(Y, \sigma) \to (X, \tau) \) is g-m_i continuous map. Hence \( f \) is a g- m_i homeomorphism.

(ii) ⇒ (iii). Let U be any minimal closed set in X. From (ii) \( (f^{-1})^c(U) \) is a g-minimal closed set in Y, so that \( f(U) \) is a g-minimal closed set in Y, \( f(U) \) is a g-minimal closed set in Y. Therefore \( f \) is a g-m_i closed map.

(iii) ⇒ (i). Let V be any maximal open set in X, so that V^c is a minimal closed set in X. From (iii) \( f(V^c) = (f(V))^c \) is a g-minimal closed set in Y, so that \( f(V) \) is a g-maximal open set in Y. Therefore \( f \) is a g-m_a open map.

Definition 2.4: A bijective mapping \( f : (X, \tau) \to (Y,\sigma) \), is said to be generalized minimal* homeomorphism (briefly g- m_i* homeomorphism) if \( f \) and \( f^{-1} \) are g-m_i irresolute maps.

Theorem 2.5: If \( f : (X, \tau) \to (Y,\sigma) \) is a bijective map, then the following statements are equivalent.

(i) Its inverse map \( f^{-1} : (Y, \sigma) \to (X, \tau) \) is g-m_i irresolute.

(ii) \( f \) is a g-m_a* open map.

(iii) \( f \) is a g-m_a* closed map.

Proof: (i) ⇒ (ii). Let V be any g-maximal open set in X, then V^c is a g-minimal closed set in X. From (i) \( (f^{-1})^c(V^c) = f(V^c) = (f(V))^c \) is a g-minimal closed set in Y, so that \( f(V) \) is a g-maximal open set in Y. Therefore \( f \) is a g-m_a* open map.

(ii) ⇒ (iii). Let U be any g-minimal closed set in X, so that U^c is a g-maximal open set in X. From (ii) \( f(U^c) = (f(U))^c \) is a g-maximal open set in Y, so that \( f(U) \) is a g-minimal closed set in Y. Therefore \( f \) is a g-m_a* closed map.

(iii) ⇒ (i). Let U be any g-minimal closed set in X. From (iii) \( f(U) \) is a g-minimal closed set in Y, so that \( (f^{-1})^c(U) \) is a g-minimal closed set in Y. Therefore the inverse map \( f^{-1} : (Y, \sigma) \to (X, \tau) \) is g-m_i irresolute map.

Theorem 2.6: If \( f : (X, \tau) \to (Y,\sigma) \) is a bijective map and g-minimal irresolute map, then the following statements are equivalent.

(i) \( f \) is a g-m_a* open map.

(ii) \( f \) is a g- m_i* homeomorphism.

(iii) \( f \) is a g-m_i* closed map.

Proof: Similar to the proof of the Theorem 2.3.
Theorem 2.7: If \( f: (X, \tau) \rightarrow (Y, \sigma) \) and \( h: (Y, \sigma) \rightarrow (Z, \eta) \) are \( g\)-\( m_i^* \) homeomorphisms, then \( h \circ f: (X, \tau) \rightarrow (Z, \eta) \) is \( g\)-\( m_i^* \) homeomorphism.

**Proof:** Let \( U \) be any \( g\)-\( m_i \) closed set in \( Z \). Since \( h \) is \( g\)-\( m_i^* \) homeomorphism, \( h^{-1}(U) \) is a \( g\)-\( m_i \) closed set in \( Y \). But \( f: (X, \tau) \rightarrow (Y, \sigma) \) is \( g\)-\( m_i^* \) homeomorphism. Therefore
\[
f^{-1}[h^{-1}(U)] = (h \circ f)^{-1}(U)
\]
is a \( g\)-\( m_i \) closed set in \( X \). Hence \( h \circ f: (X, \tau) \rightarrow (Z, \eta) \) is a \( g\)-\( m_i \) irresolute map.

Again let \( V \) be any \( g\)-\( m_i \) closed set in \( X \). Since \( f \) is a \( g\)-\( m_i^* \) homeomorphism,
\[
f(V) = (f^{-1})^{-1}(V)
\]
is a \( g\)-\( m_i \) closed set in \( Y \). But \( h: (Y, \sigma) \rightarrow (Z, \eta) \) is a \( g\)-\( m_i^* \) homeomorphism. Therefore
\[
[(h^{-1})^{-1}(f^{-1})^{-1}](V) = [(h^{-1})^{-1} \circ (f^{-1})^{-1}](V)
\]
is a \( g\)-\( m_i \) closed set in \( Z \). That is
\[
[(f^{-1} \circ h^{-1})^{-1}](V) = [(h \circ f)^{-1}]^{-1}(V)
\]
is a \( g\)-\( m_i \) closed set in \( Z \). It follows that \( (h \circ f)^{-1} \) is \( g\)-\( m_i \) irresolute. Hence \( h \circ f: (X, \tau) \rightarrow (Z, \eta) \) is \( g\)-\( m_i^* \) homeomorphism.

**REFERENCES**


