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Applications and Variants of Ant Colony Optimizationin Design of Digital Filters: AReview

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ABSTRACT:
In this paper, a comprehensive review on the various applications and advances in design of digital filters using ant colony optimization (ACO) is presented.Also, various variants of ACO algorithm are discussed. Here, the performance of these filters is reviewed and compared in terms of various parameters like computation time, errors and ripples etc.

Keywords: Digital Filters, Optimization, Genetic Algorithm, Particle Swarm Optimization, Ant Colony Optimization.

1.INTRODUCTION

Digital filters can be described as a network by which the shapes of waves, amplitude, phase, frequencycan be changed in a desired manner. Basically, digital filters play essential role in broad range of signal processing applications that include digital TV's,mobile phones and equipment'sthat are used in measurement of physical signals [1]. Digital filters have linear and non-linear phase responses.Digital filters arebetterfrom their analog counterpart due to their wide range of applications and best performance. The advantages of digital filters over analog filters are high accuracy,small physical size and reliability [2]. Digital filters are mainly divided in to two types known as finite impulse response (FIR) i.e. non-recursive filters and infinite impulse response (IIR) i.e. recursive filters [4].

1.1 Finite Impulse Response filter

These filters have a polynomial with transfer function in Z-plane and all FIR filters has only zeros and have linear phase.It is used to find out magnitude and frequency response. The z- transform of FIR filters that have N-points as follows:

$$H(z) = \sum_{n=0}^{N-1} h(n)z^{-n} \qquad (1)$$

These filters are broadly used in applications of digital signal processing due to stability and linearphase properties [5-6]. FIR filters require no feedback.

1.2 Infinite Impulse Response Filter

IIR filters are nonlinear and multimodal. As the order of the digital IIR filterincreases,itsstructure becomes more complex and computational cost increases[3]. The designing of IIR filter depends on the magnitude response of filter.The filter is causal if the impulse response h[n] = 0 for n < 0, it impliesthat h[n] is not symmetrical. So, an IIR filter does not have linear-phase characteristics. IIR filter can be described in the form of transfer function as follows:

$$H(s) = K\frac{(s-z_1)(s-z_2)(s-z_3)\ldots\ldots}{(s-p_1)(s-p_2)(s-p_3)\ldots\ldots\ldots} \qquad (2)$$

The IIRfilter is characterized by its poles(p_1, p_2, p_3…) and its zeros (z_1, z_2, z_3…). FIR filter's delay characteristics are much better but they require more storage. IIRfiltersare difficulttodesign as compared to FIR filters [7].

1.3 DesignMethods

Various design methods for both digital FIR and IIR filters are described as follows:

a) **Digital FIR Filter**

In this section,various methods for designing FIR filters are presented. There are various methods which are used to design the FIR filter as follows: - 1). Symmetric and Antisymmetric FIR Filters 2).Windows method 3). Frequency-sampling method 4). Optimum Equiripple Linear-phase FIR filter.

*1). Design of Symmetric and Antisymmetric Digital FIR Filters:*DigitalFIR filter having length M,input*x(n)*&output *y(n)* is defined by difference equation:

$$y(n)=\sum_{k=0}^{M-1} b_k x(n-k) \qquad (3)$$

*2). Design of DigitalFIR filters using Windows:*It is amethod in which the frequency response has desired frequency response$H_d(\omega)$ and definestheunitimpulse response $h_d(n)$. Although, $h_d(n)$ is dependent on $H_d(\omega)$ by Fourier Transform:

$$H_d(\omega)=\sum_{n=0}^{\infty} h_d(n)\, e^{-j\omega n} \qquad (4)$$

$$h_d(n)=\frac{1}{2}\int_{-\pi}^{\pi} H_d(\omega)\, e^{j\omega n} d\omega \qquad (5)$$

In this, Bartlett, Blackman, Hamming, HanningandKaiser windows are used.

*3).Design of Linear-phase Digital FIR Filter by Frequency-sampling method:*This method is usedindesigning FIR filters, desired frequency response$H_d(\omega)$ is calculated with a pair of same difference frequencies:

$$\omega_k = \frac{2}{M}(k+\alpha), \quad k=0,1,...,(M-1)/2 , M=ODD$$

$$k=0,1,..,M/2-1 , M=EVEN, \alpha =0 \text{ and } ½, \qquad (6)$$

4). Design ofOptimumEquirippleLinear-phase Digital FIR filter: The frequency sampling method and window method are relatively elementary techniques for designing FIR filters. To define optimum equiripplemethod[8], assuming the design of a digitallowpassfilter with the passband edge frequency ω_p and the stopband edge frequency ω_s. The filter frequency response satisfies the condition:

$$1-\delta_1 \le H_r(\omega) \le 1+\delta_1 , \quad |\omega| \le \omega_p \qquad (7)$$

$$-\delta_2 \le H_r(\omega) \le \delta_2 , \; |\omega| > \omega_s \qquad (8)$$

b) **Digital IIR Filter**

In this section,various methods of designing of IIR filter are described. In this, magnitude and phase response characteristics are specified. Here, IIR filter designing methods are as follows: - 1).IIR Filter Design by Approximation of Derivatives2). IIR Filter Design by Impulse Invariance 3). IIR Filter Design by Bilinear Transformation.

*1).Digital IIR Filter Design by Approximation of Derivatives:*It is simplest and easiest method in which an analog filter is converted into digital filter by using a differential equation and this method is mainly used for solving a linear constant-coefficient differential equation as follows: -

$$\frac{dy\,(t)}{dt}\Big|_{t=n} = \frac{y(nT)-y(nT-T)}{T} = \frac{y(n)-y(n-1)}{T} \qquad (9)$$

*2).DigitalIIR Filter Design by Impulse Invariance:*In this method,theobjective is to design an digital IIR filter having *h(n)*: -

$$h(n) = h(nT) \quad n = 0,1,2,3...... \qquad (10)$$

3). Digital IIR Filter Design by Bilinear Transformation – This method [8]islinked to the trapezoidal formula. Assume an analog filter with system function: -

$$H(s) = \frac{b}{s+a} \qquad (11)$$

In order to overcome the limitations, which exists in the traditional IIR filters design methods, IIR filters stimulated the requirement of developing innovative optimization techniques.Here, it has larger compromising in shaping and their magnitude response.

2. OPTIMIZATION TECHNIQUES

- **Need of optimization-** The traditional non-optimization methods for filter implementation suffers from the trouble of inefficient frequency response and also it requires analog to digital conversion[12]. Various conventional gradient basedoptimization techniques are not able to solvenon-differential function. Optimization process has rule to adjust the process that is used to optimize some defined set of parameters without breaking some constraint. The main goalisdecreasing cost and increasing throughput or efficiency. It is one of the biggest tool of quantitative in industrialdecision-making.To optimize the process, the aim is to maximize one or more of the process specifications, while having all others within their constraints.
- **UnimodalOptimization –** In this,unimodality means possessing a different mode. Mostly, unimodality means there is only a single largest value, defined and some mathematical object[23].
- **Multimodal optimization-** In this, multimodal optimization givesmultiple best answers. Those could all be globally best and there can be combination of globally good and locally good solutions[23]. Obtaining all the multiple solutions is the aim of a multi-modal optimizer.
- **Genetic Algorithm**-Genetic algorithm (GA)is meta heuristic process. This process is produced by natural selection which is related to thebigger section of evolutionary algorithm.GA is mainly used for obtaining high-quality results for problems depending on functions like crossover, selection and mutation [13].
- *Chromosome representation*-It is an easiestmethod that showssingle chromosome like a bit string. It has numeric parameters which could be shown by integers, through thiscanutilize representation of floating point. This floating point shows evolutionary programming and natural to evolution schemes[17].
- *Elitism*– It is a practical type of common procedure of building a modern population that provides best organism from the current generation to hold over to the next, unaltered.It is a schemethat is calledelitistselection[18].
- *Parallel implementations* –These are two processes of genetic algorithm as follows: -

i) Coarse-grained- This is genetic algorithm with parallel property. It considers population upon migration of separated between the nodes and single of the computer nodes.

ii) Fine-grained- It considers randomly onsingle processor node that behaves with neighboring individuals for reproduction and selection [19].

- *Adaptive GAs*-Adaptive genetic algorithm that has promising variantandsignificantof genetic algorithms. It has probability of mutation and crossover largely find the convergence speed and accuracy from genetic algorithm get[20].

Limitations–This algorithm generally converge towards local optima. It could be arbitrary points instead the global optimum [15].Operating on dynamic data sets is hardly to get solutions. Genetic algorithm could not resolve obstacles that includes exclusively fitness function has individualmeasure of wrong or right and it has no convergence path for solution[16].

- **Particle Swarm Optimization-** This optimization technique was discovered by Eberhart and Kennedy [14]. It is generated by a population of random solutions and individual potential solution is assigned a random velocity. These potential solutions are known as particles and after that these are ‘flown’ through the space of problem that are assorted with best solutions or fitness get. The fitness value is saved and this value is known as ‘pbest’

Limitations- This method suffers from the partial optimism that leads to decrease in its speed and the direction[20]. It may not work for the obstacles of scattering and optimization[20].

- **Ant Colony Optimization-**This technique is discovered by MARCO DORIGO (1991).It is used to calculate shortest path between source and destination. The first ant drifts randomly until it finds the food source (F) then it goes back to nest(N) depending on PHEROMONE TRAIL [9]. PHEROMONE TRAIL, it is

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a chemical substance that is released by ants when it goes into search of food.Shortest path is discovered by pheromone trails. Pheromones shows in some path the common storage. It remains extremely elementary [10].Combinatorial optimization is used to calculate optimum solutions from finite number of solutions[21].It works on the domain of that optimization problems that has set of appropriate solutions that will discrete. There could lesser to discrete. It has aim is to find the best solution[22]. The examples of this optimization are minimum spanning tree problem and travelling salesman problem[22].

- **Components of Ant colony optimization**
- **Pheromone:** It has chemical substance that could be discharged from ants. Artificialantsthat have pheromones which is used to calculate optimum solution in various types of problems that are not fixed and have nature of combinatorial. Those pheromone trails haveexplosive behavior in nature[11].These could evaporate aftera certain amount of time.
- **Initial value of Pheromone:** When ant travels on the paths, it gives the quantity of Pheromone. Starting values of pheromone could lay 1/0. Here, it is 1, the value of starting pheromone after that the rate of evaporation would be assigned path and in starting it isevaporated and after that the pheromone will be place on way. Here, it is 0,it has placing property would stimulateafter that the pheromone rate of evaporation would decrease the quantity of pheromone.
- **Stigmergy:** It has not a direct communication among ants through pheromone set over the way. This is applicableupon natural ants that it hides pheromone upon the way and successor ant would note the amount of pheromone quantity after that determine to which way it requires to travel. It is type of not direct communication; known as Stigmergy**.**
- **Pheromone Evaporation Rate:** It is related to the pheromone decay in unit time. Here,evaporation is more required in artificial ants to ignore ways that are not optimal to increase the convergence. Now, evaporation rate isshown by Δ.
- **Pheromone Decay Coefficient:** This has a constant value that is related the decay of the pheromone.
- **ACO PSEUDO CODE-** Here, we describe ACO pseudo code [27]as follows:-

Random initialization of pheromone value

Do

For each iteration

For each ant

Compute probability P according to probability equation i.e.

$$P_{n,k}^{1m}(t) = \frac{\tau^{l_i}{}_{n,k}(l)}{\tau^{l1}{}_{n,k}(t) + \tau^{lu}{}_{n,k}(l) + \tau^{l\bar{1}}{}_{n,k}(l)} \qquad (12)$$

Determine the Pmax

End

Compute OF

End

Deduce the best OF

Update pheromone values

according to pheromone updation equation i.e.

$$\tau^{l_i}{}_{n,k}(t+1) = \rho\tau^{l_i}{}_{n,k}(t) + \sum_{u=1}^{\sigma-1} \Delta\tau_{n,k}^{lm,u}(t) + \Delta\tau_{n,k}^{lm,*}(t) \qquad (13)$$

End

Report the best solution

END

The main aim of ant colony optimization is to optimize problems that aretoughin nature. Several types of problems have been solved using ant colony optimization.Some are NP hard problems that could be resolved only in polynomial time.

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Figure 1.2 illustratestheflowchart ofACOalgorithm from that it describes optimal solution which is used for NP problems.

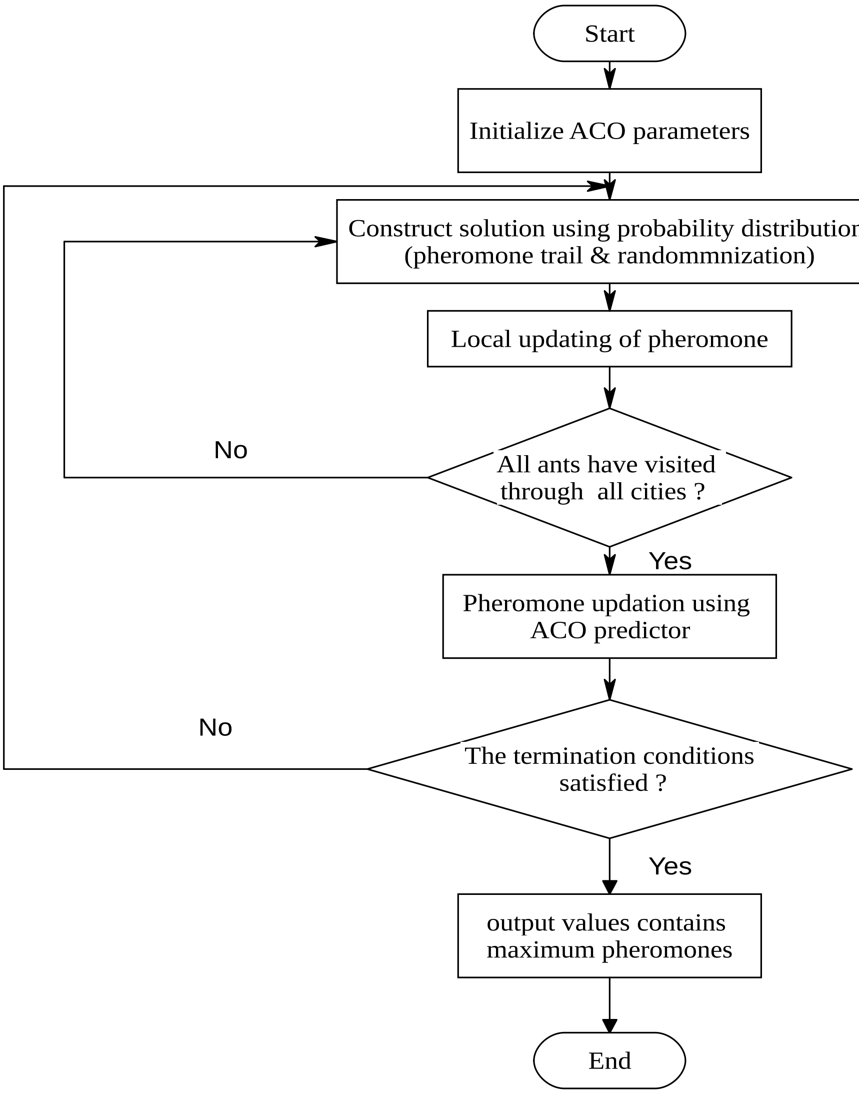

Fig1:Flowchart of ACO algorithm[9]

3. VARIANTS OF ACO

- **Ant System**-It was discovered by Dorigoin 1991. The first implementation is used in the Travelling Salesman Problem[10]. It is an algorithm nature inspired by foraging behavior of ants. It is used for local updation. It takes more computation time at execution.
- **Ant Colony System-** Itwasdiscovered by Dorigo in 1991. It is used to find a shortest path between source and destination. It is also an algorithm nature inspired by foraging behavior of ants. It is used for local and global updation for both. It takes less time at execution.
- **Max-Min Ant System-** It is a solution that is builtin the equivalent manner asAntsystem[10]. Modifying made in max-min ant system upon ant system are aggregate 3 in number. Starting canexploitthe best solutioncalculatedduring the iteration, after every iteration only one ant would be admitted to update the pheromone on the path that it covered.

- **Quantum Ant Colony Optimization-** Themotive has to define the quantum rotation gate and Q-bit representation into ACO. It isusedtoimplement the hyper-cube framework and develop a discrete binary ACO algorithm[10].
- **Cunning Ant System-**It is differentfrom elementary ant colony optimization. Here, elementary ant colony optimization algorithms ants would develop the solution based on the present pheromone trailt_{ij}(t)[10].
- **Cooperative Genetic Ant System-** It is equal to a hybrid algorithm that has combination of Ant System and Genetic Algorithm that showsthat it is Cooperative Genetic Ant System (CGAS). It is method that exists in both Ant system and genetic algorithm concurrently and cooperatively[10]. It returns an improve chance in attaining global solution of travelling salesman problem.
- **Modified ACO-** This method explains regression testing. This testing changes existing and unchanged code would continue to behave as expected after the changes have been incorporated. This m-ACO gives modified form of ant colony optimizationand in the sense that here real ants find a food source and it returns food to its nest and then deposit food in the nest and then goes to the same food source again until the food sourcefinishes[24].
- **Adaptive multimodal Continuous ACO-**It has multiple optimal solutions. It shows many designs with similarity performance that are utilized in practical applications and also in demand that makes multiple choices. This optimization solves complex problems. They still cannot locate all the known global optima when a large number of local optima exist[25].

4. APPLICATIONS OFVARIOUSACO

Ant Colony Optimization has various applications in the design of digital filters which are used in many other applications as follows: -

- **Design and Optimization of IIR Digital Filters with Non-standard Characteristics Using Continuous Ant Colony Optimization Algorithm-** Here, design and optimization of IIR filters using ACO is studied. It is used in amplitude equalizers. In this, three stable IIR filters are designed with non-standard amplitude [26].
- **Ant Colony Optimization for Optimal Low-Pass Butterworth Filter Design-** Analog filters depend on discrete components and gives strong values.Digital filters generally give stability and linear response. Antcolony optimization gives optimal solutions when it is usedwithButterworth filter. It has high accuracy and having short execution time. It has also good convergence[27].
- **Optimization of LMS Algorithm for Adaptive Filtering using Global Optimization Techniques-** Here, IIR filters are optimized using gradient based optimization techniques. It is used for step size representation by using ant colony optimization and particle swarm optimization. It also removes multi-modal errors and gives average error values[28].
- **A Hybrid PSO/ACO Algorithm for Discovering Classification Rules in Data Mining-** Here, in this set of attribute values are used. In this hybrid of PSO/ACO is explained. It gives much simpler and desirable results. It also used domain benchmark datasheets in the classification [29].
- **An Approach to Enhance Performance of Kaiser Window Based Filter-** In this signal get affected by unwanted components and adverse but inevitable. Here, removal of unwanted components is termed as removal of noise. It is mostly utilized in hospital and diagnostic centers from last many decades. Kaiser window has innovative nature by inspired algorithm[30].
- **Image Enhancement by Adaptive Filter with Ant Colony Optimization-** The goal is to perform image enhancement by using adaptive filter with ant colony and to removethe attributes and makes them more suitable. It improves the visual quality of image [31].
- **An Acceleration of Designing FIR Filters with CSD Coefficients Using ACO –** In this paper**,** combinatorial problem is solved and with using CSD coefficient that is used to remove non-zero digits and reduction in computation time. It gives an optimal solution with using FIR filters. Due to its properties, it has stability and linear phase[32].

- **An ACO Approach for Design of CSD Coefficient FIR Filters** – FIR filter gives linear phase characteristics and has stability. Due to its properties, it is used with ACO with CSDcoefficient.CSD is used to remove non- zero digits and reduction in computation time[33].
- **Design of FIR Filters with Discrete Coefficients Using Ant Colony Optimization** –In this, pheromone update is performed. Here, the use of FIR filters with discrete coefficients using ACO is presented to reduce computation time. It also reduces the maximum errors[34].

5. CONCLUSION

In this paper, various types of applications of ACO in the designing of digital filtersarereviewed. From these applications, various variants of ACO and the performance of these filters arestudied and compared in terms of various parameters like computation time, errors etc.

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