

# A Firefly based Controller Design using Approximate Model Matching

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## ABSTRACT

*This paper proposes an approximate model matching technique using firefly algorithm to obtain the PID parameters. It uses different performance indices to minimize objective function for the firefly algorithm. A sample SISO system with step response has been taken in the paper and it can be seen that Integral Time-weighted Square Error (ITSE) gives us the best minimized value for our SISO system. In future, modifications can be made in the current algorithm used to obtain more accurate results.*

## Keywords

*Approximate model matching (AMM), Firefly Algorithm (FA), Controller design, Performance Indices.*

## I. INTRODUCTION

The modern advances in technology and decision making caused the engineering problems get harder and more complex. This created the need for intelligent control system to manage complexity and uncertainty through the design of system and processes enabling high precision and accuracy. Control system design requires knowledge of, the plant which describes the mathematically modeled behavior of a system and the output [1]. Industries today use Proportional-Integral—Derivative (PID) Controllers or modified PID controllers as feedback control for most automatic process control applications.

Traditionally, tuning of controllers was done by trial and error approach using methods proposed by Ziegler-Nichols [6] and Cohen-Coon [7]. But these methods were not as accurate and occasionally yielded poor performance in practice [2].

The higher order controllers are found to be fragile which may even lead to instability for very small changes in the controller coefficients. There are two approaches for control design [12],

- ) EMM
- ) AMM

Model matching control technique plays an important role in controller synthesis [8]. The main aim of Exact Model Matching (EMM) controller is to design a closed loop transfer function model that exactly follows the reference plant model [9]. The applicability of EMM in case of practical cases appears to be limited unless a higher order dynamic controller is used. Prompted by the need to implement simple systems and use simple reference models some authors have recently developed Approximate model matching (AMM) technique, with this technique designers have greater freedom to choose the structure of controller and reference model [11]. Moreover, some metaheuristic techniques were used for the optimization of the parameters.

In the past few years metaheuristic algorithms are being used for optimizing control systems as they efficiently deal with nonlinear optimization problems like traveling Salesman Problem, Machine learning,

Pattern Recognition and many more.. In this paper an efficient tuning approach is proposed to find the optimal PID parameters. The approach is based on Firefly Algorithm (FA) introduced by Yang [3].

The Firefly Algorithm (FA) is based on the flashing characteristics of the fireflies and the flashing light is associated with the objective function to be optimized which makes it possible to formulate new optimized model [3]. In this paper, parameters of PID controller are tuned by applying FA and Model Matching method has been utilized to minimize the objective function for firefly Algorithm.

To achieve minimum performance index, the following indices used are Square Error (SE), Integral Square Error (ISE), Integral Absolute error (IAE), Integral time weighted square error (ITSE), and integral time weighted absolute error (ITAE). And a comparison has been done between the above indices to find the best value of the PID parameters.

The paper is organized as follows. Section II describes PID parameter and presents mathematical model for a system. In Section III the performance indices have been described. In Section IV describe the basis of firefly algorithm. Section V includes numerical simulation and comparison. Finally, some conclusions are drawn in Section VI.

## II. MATHEMATICAL MODELLING OF PID CONTROLLER

Proportional-Integral-Derivative (PID) controller minimizes error over time by adjustment of control variables. The control variable is defined as

$$C(s) = K_p + \frac{K_i}{s} + sK_d \quad (1)$$

where  $K_p$ ,  $K_i$  and  $K_d$  denotes nonnegative coefficients of the proportional, integral and derivative terms [4].

The proportional controller reduces the rise time, the integral controller reduces the steady state error for a constant or step input and a derivative control increases the stability of the system in addition reducing overshoot and improving transient response. The PID controller finally increases the speed of the overall control system response. The coefficients  $K_p$ ,  $K_i$  and  $K_d$  are thus to be tuned using Firefly Algorithm.

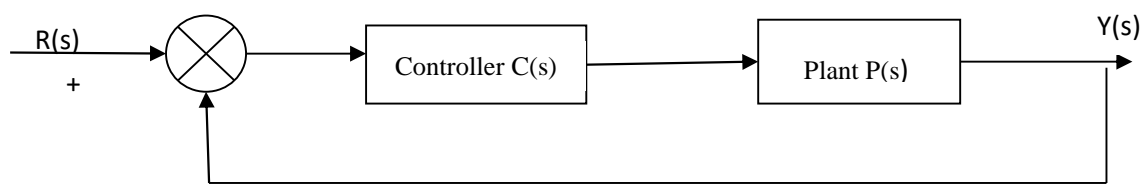


Fig 1: Standard unity feedback configuration

Transfer Function of above system is :

$$F(s) = \frac{Y(s)}{R(s)} = \frac{C(s)P(s)}{1+C(s)P(s)} \quad (2)$$

Where  $Y(s)$  is output response,  $R(s)$  is input response,  $C(s)$  is controller transfer function and  $P(s)$  is plant transfer function.

## III. MODEL SELECTION AND PERFORMANCES INDICES

The reference model transfer function should be chosen to have a sufficiently rapid response; on the other hand it should keep high frequency gain of the controller. The reference model might be required to satisfy some of the following design specification:

- ) The time domain specification, e.g., the rise time, overshoot, settling time and steady state error.
- ) The frequency domain specifications, e.g., the bandwidth, the cutoff rate, gain margin and phase margin.
- ) The complex domain specifications, e.g., the damping ratio, damping factor and damped natural frequency.

The reference model selection has been done using the technique defined by Chen [5].

$$M(s) = \frac{\sum_{i=0}^k d_i s^i}{\sum_{i=0}^l c_i s^i}, \quad k \leq l \quad (3)$$

Let the model be specified as

$$M(s) = \frac{d_0 + d_1 s}{c_0 + c_1 s + s^2} \quad (4)$$

For steady state matching  $d_0 = c_0$ . Then

$$M(s) = \frac{c_0 + d_1 s}{c_0 + c_1 s + s^2} \quad (5)$$

Let the desired closed loop specifications that  $M(s)$  has to satisfy be

Velocity error constant:  $k_v$

Crossover frequency:  $\omega_c$

Damping ratio:  $\xi$

The parameters  $c_0$ ,  $c_1$  and  $d_1$  are determined by solving the following equation [5].

$$\omega_c^2 c_1^2 - 2\omega_c^2 c_1 d_1 - c_0^2 = -\omega_c^4 \quad (6)$$

$$c_1 - \left(\frac{c_0}{k_v}\right) - d_1 = 0 \quad (7)$$

$$c_1^2 - 4c_0\xi^2 = 0 \quad (8)$$

For given values of  $\omega_c$ ,  $\xi$  and  $k_v$  using the above equations we can define a reference model  $M(s)$  in complex domain.

In exact model matching (EMM), we have

$$F(s) = M(s) \quad (9)$$

In some cases, it is found that this design method may not lead to the simplest form of compensator. The resulting controller may be of higher order and unstable hence EMM is not applicable. In the case of approximate model matching (AMM) Eqn. (2) is approximately satisfied, i.e.

$$F(s) \approx M(s) \quad (10)$$

$$M(s) = \frac{Y_r(s)}{R_r(s)} \quad (11)$$

Where,  $Y_r(s)$  is Output response of reference model and  $R_r(s)$  is input step response of reference model.

$$e = Y(s) - Y_r(s) \quad (12)$$

#### A. Performance indices

To optimize the performance of a closed loop control system we can try to adjust the control system parameters to maximize or minimize some performance index.

**Table 1: Description of Performance Indices and their Characteristics**

Performance indices	Characteristics
$S_e = \sum_{t=0}^N e_t^2(t)$	It measures the total deviation of the response values from the fit to the response values.
$I_s = \int_0^T e^2(t) dt$	The step response of the system designed is fast but oscillatory in steady state error is zero
$I_L = \int_0^T  e(t)  dt$	It has reasonable damping and an acceptable transient response behavior.
$I_{ts} = \int_0^T t e^2(t) dt$	It has less overshoot, shorter settling time and better selectivity
$I_{ts} = \int_0^T t  e(t)  dt$	It has zero steady state error and lesser oscillations as well as smaller overshoot. Further it has good selectivity.

#### IV. FIREFLY ALGORITHM

Xin-She Yang developed the firefly algorithm using the flashing characteristics of fireflies. Following are the assumptions made in the firefly algorithm [3].

1. All fireflies will be attracted to every other firefly regardless of the sex, i.e., to say that they are unisex.
2. The attractiveness and brightness decrease as the distance increase and are also proportional to each other. The less bright will be moving towards the brighter one. It will move randomly if there is no brighter one.
3. The brightness of firefly is determined or affected by the shape of the objective function.

In the firefly algorithm, there are two important issues: the variation of light intensity and formulation of the attractiveness. The light intensity at a particular distance 'r' from the light source obeys inverse square law.

$$I(r) = \frac{I_s}{r^2} \quad (13)$$

where  $I_s$  is the intensity of the light source. Light is also absorbed so we should allow the attractiveness to vary with the degree of absorption. In order to avoid the singularity at  $r=0$  in the expression above.

Combined effect of the laws can be approximated as

$$I(r) = I_0 e^{-\gamma r^2} \quad (14)$$

where  $I_0$  is the original light intensity and  $\gamma$  is the absorption co-efficient.

The attractiveness of the firefly can be defined as

$$\beta = \beta_0 e^{-\gamma r^2} \quad (15)$$

where  $\beta_0$  is attractiveness at  $r=0$  [3].

The movement of a firefly  $i$  is attracted to more another attractive firefly  $j$  is determined by

$$x_i = x_i + \beta_0 e^{-\gamma r_{ij}^2} (x_j - x_i) + \alpha \epsilon_i \quad (16)$$

where second term is due to attraction and third is randomization factor.

For a maximization or minimization problem the brightness can simply be proportional to the value of the objective function based on these rules, the basic steps of firefly algorithm can be summarized as the pseudo code shown in figure 2.

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Objective function  $f(x)$ ,  $x=(x_1, \dots, x_d)^T$ 
Generate initial population of fireflies  $x_i(i=1,2,\dots,n)$ 
Light intensity  $I_i$  at  $x_i$  is determined by  $f(x_i)$ 
Define light absorption coefficient
while( $t < \text{MaxGeneration}$ )
  for  $i=1:n$  all  $n$  fireflies
    for  $j=1:n$  all  $n$  fireflies(inner loop)
      if( $I_i < I_j$ ), move firefly  $i$  towards  $j$ ;
    end if
    vary attractiveness with distance  $r$  via  $e^{-r}$ 
    evaluate new solutions and update light intensity
  end for j
end for i
  rank the fireflies and find the current global best  $g^*$ 
end while
postprocess results and visualization.

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**Fig2. Pseudo code for firefly algorithm**

## V. NUMERICAL SIMULATION AND COMPARISON

A simple plant has been considered as follows [10].

$$G_p(s) = \frac{3}{s^2 + 4s + 3} \quad (17)$$

Reference model for design specifications  $\zeta=5.0$ ,  $\omega_n=0.707$  is

$$M(s) = \frac{4.2}{s^2 + 7.0} \frac{s+2}{s+2} \quad (18)$$

The PID controller is taken as

$$C(s) = K_p + \frac{K_i}{s} + sK_d \quad (19)$$

The values of  $K_p$ ,  $K_i$  and  $K_d$  are found using the firefly algorithm where, the error obtained after approximate model matching has been optimized. The values of the parameters assumed for firefly algorithm are given in Table 2.

**Table 2: Firefly Algorithm Parameters**

Parameters	Experimental Values
Population size	20
No. of iterations	50
(Mutation coefficient)	0.2
(Attraction coefficient)	0.2
(Absorption coefficient)	1.0

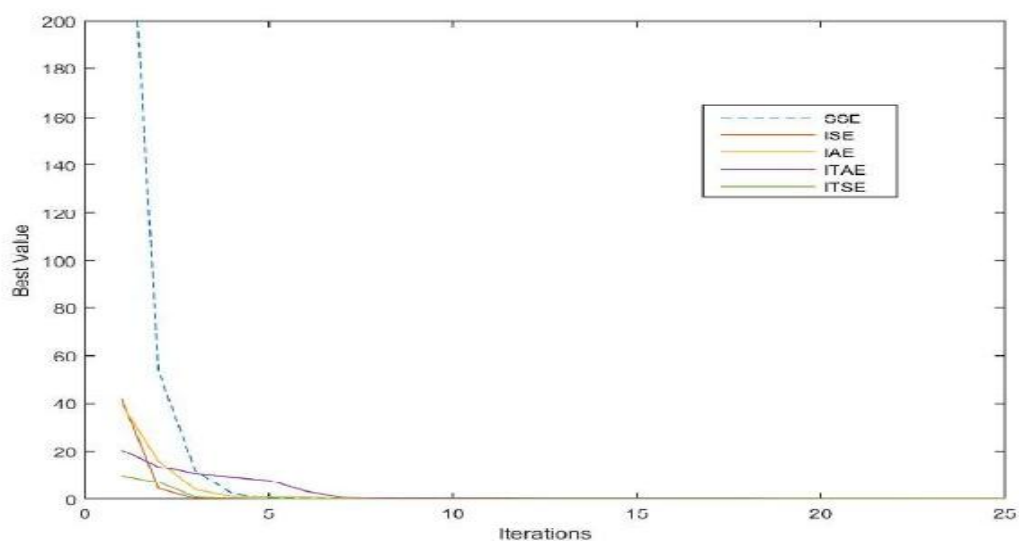
Table 3 provides a comparison between the  $K_p$ ,  $K_d$  and  $K_i$  values obtained for different performance indices. Table 4 describes the optimized value of the objective function for each performance index.

**Table 3: Parameter for different performance indices of test system**

PID Parameters	SSE	ISE	IAE	ITAE	ITSE
$k_p$	1.3599	1.3432	1.2288	1.025	1.2137
$k_i$	0.9261	0.9526	0.9897	0.9999	0.98528
$k_d$	0.0116	0.0078	0.0014	0.0000	0.00267

**Table 4: Best value after 50<sup>th</sup> Iteration**

	SSE	ISE	IAE	ITAE	ITSE
<b>Best Value</b>	0.1267	0.012584	0.18576	0.11713	0.010824

**Fig 2: Convergence characteristic curve of performance indices**

## VI. CONCLUSION

The problem of controller design of continuous-time systems has been considered for SISO system. A new time-domain FA-based method has been proposed that uses the concept of approximate model matching. Several time-domain performance criteria have been successfully minimized to tune the parameters of PID controllers. The method developed in this paper use only output(s) for feedback purpose, and hence the controller synthesized is easy to implement, more reliable as there are lesser number of signals to be measured for feedback purpose. Further both the cost and complexity of hardware implementation is also less.

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