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# Uncertainty and Disturbance Estimator Based Sliding Mode Control of 2<sup>nd</sup> Order System

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## ABSTRACT

*This work deals with the issue of control of an uncertain system by using uncertainty and disturbance estimator (UDE). A typical second order system is considered for illustration. The results may be extended to a real life application. UDE is used to estimate the uncertainty and the control law ensures robust performance. The proposed technique is validated through simulation in MATLAB. The effectiveness of UDE based control is shown for different types of disturbances.*

## Keywords

*Uncertainty and disturbance estimator (UDE), Robust control, Disturbance estimation.*

## I. INTRODUCTION

In most of the practical systems uncertainties are always present. Uncertainty includes unmodeled dynamics and external disturbances. Disturbance signals and dynamic perturbations are two varieties of uncertainties. Input and output disturbance, sensor noise and actuator noise, etc are included in disturbance signals. A mathematical model of any real system is always just an approximation of the real physical system. The difference between actual and mathematical model includes unmodeled dynamics (usually high-frequency), neglected nonlinearities in the modeling, effects of reduced-order models, and system parameter variations due to environmental changes. The stability and performance of a control system may adversely be affected due to these uncertainties. With growing interest in high-precision control, utilization of disturbance rejection technique is generally required in the controller design. During the past decades, Time delay control (TDC) [3] is used for system with unknown dynamics. It is based on the assumption that a continuous signal remains unchanged during a small enough period, the past observation of uncertainties and disturbances is used to modify the control action directly. But it has some limitations such as, presence of oscillations in control system and due to delay system becomes unstable.

To overcome this issue recently an uncertainty and disturbance estimator (UDE)-based control method was proposed in [1]. It has good capability of uncertainty and disturbance rejection and reference tracking. UDE is considered as replacement of time delay control (TDC). UDE-based control method is based on assumption that a continuous signal can be approximated when it passes through an appropriate filter. Notable feature of UDE is that it is not affected by modelling inaccuracies and does not require apriori knowledge of disturbances, except the information about the bandwidth, during the design process (but needed for the analysis of stability).

Disturbances can be external disturbances or internal parameter variations. UDE based control law can effectively tackle all such uncertainties. UDE technique is successfully applied to diverse system like non-affine nonlinear system [4], robust input-output linearization [5], robot manipulator [7], voltage control of DC-DC power converter [8], robust control of electric motor drive [9], control of Unmanned Aerial vehicles

(UAVs) [10], current control scheme for PMSM drives[11], robust control of single axis gimbal platform for micro air vehicles (MAV) [12] , trajectory tracking control of piezoelectric stages [13] and nonlinear state delay system [14]. This paper demonstrates the usefulness of the elegant technique by considering a simple academic example. However, the results may be obtained for complex system as well.

The paper is organized as follows: section II presents derivation of UDE based control law. The performance of proposed control scheme is demonstrated through simulation results in section III. And finally section IV is the conclusion.

## II. UDE-BASED CONTROL LAW

### A. System description

The  $n^{\text{th}}$  order system under conditions of uncertainty and disturbance can be expressed as

$$\dot{X} = (A + \Delta A)X + (B + \Delta B)u + d \quad (1)$$

$$\dot{X} = A + B + B(X, t) \quad (2)$$

Where,  $X$  is state variable ( $n \times 1$ ),  $u$  is control input,  $A$  is known state matrix ( $n \times n$ ),  $B$  is known input matrix ( $n \times 1$ ). Here  $\Delta A$  is uncertainty in plant matrix and  $\Delta B$  is uncertainty in input matrix.  $D(X, t)$  is the lumped disturbance.

### B. Control design

The sliding surface equation is,

$$\sigma = C^T * X$$

where,  $C^T$  is coefficient matrix.

Now,

$$\dot{\sigma} = C^T * \dot{X}$$

From equation (2)

$$\dot{\sigma} = C^T A X + C^T B u + C^T B D(X, t) \quad (3)$$

Control input  $u$  is splits in two term  $u_e$  and  $u_n$

Therefore equation (3) become,

$$\dot{\sigma} = C^T A X + C^T B u_e + C^T B u_n + C^T B D(X, t) \quad (4)$$

From equation (4),

$$u_e = -(C^T B)^{-1} C^T A X \quad (5)$$

After putting equation (5) in (4) we get,

$$\begin{aligned} \dot{\sigma} &= C^T A X - C^T B [(C^T B)^{-1} * C^T A X] + C^T B u_n + C^T B D(X, t) \\ D(X, t) &= (C^T B)^{-1} \dot{\sigma} - u_n \end{aligned}$$

(6)

In other words, the unknown dynamics and the disturbances can be observed by the system states and the control signal. However, it cannot be used in the control law directly. Assume that  $G_f(s)$  is a strictly proper low-pass filter with unity steady-state gain and broad enough bandwidth, then,  $D(X,t)$  can be accurately approximated by,

$$\hat{D}(X,t) = G_f(s)\{D(X,t)\} \quad (7)$$

Select

$$u_n = -\hat{D}(X,t) - (C^T B)^{-1} K \sigma$$

From equation (6) above equation rewritten as,

$$u_n = -[(C^T B)^{-1} \dot{\sigma} - u_n] G_f(s) - (C^T B)^{-1} K \sigma \quad (8)$$

$$u_n = \frac{-(C^T B)^{-1} \dot{\sigma} G_f(s)}{(1 - G_f(s))} - \frac{(C^T B)^{-1} K \sigma}{(1 - G_f(s))} \quad (9)$$

Assume that the frequency range of the system dynamics and the external disturbance is limited by  $\omega_f$  then,  $G_f(s)$  can be chosen as a first order low-pass filter,

$$G_f(s) = \frac{1}{1 + s\tau}$$

Where,  $\tau$  is first order filter time constant .

$$\frac{1}{1 - G_f(s)} = \frac{1}{s\tau} + 1 \quad (10)$$

And now ,

$$\frac{G(s)}{1 - G_f(s)} = \frac{1}{s\tau} \quad (11)$$

which means that an integral action is included in the controller.

Considering (10) and (11), the UDE-based control law (9) can be rearranged as

$$\begin{aligned} u_n &= -(C^T B)^{-1} \dot{\sigma} \frac{1}{s\tau} - (C^T B)^{-1} K \sigma \left[ \frac{1}{s\tau} + 1 \right] \\ u_n &= -(C^T B)^{-1} \frac{\sigma}{\tau} - (C^T B)^{-1} K \frac{\int \sigma d}{\tau} - (C^T B)^{-1} K \sigma \end{aligned} \quad (12)$$

The final UDE based control law using (5) and (12) is given as

$$u = u_e + u_n$$

$$u = -(C^T B)^{-1} C^T A X - (C^T B)^{-1} \frac{\sigma}{\tau} - (C^T B)^{-1} K \frac{\int \sigma dt}{\tau} - (C^T B)^{-1} K \sigma \quad (13)$$

### III. SIMULATION RESULTS

The simulation was carried out to test the performance of control (13) using UDE for the plant in (2) for 5 seconds using SIMULINK software.

The system matrices are  $A=[0 \ 1; -2 \ -3]$ ,  $B=[0 \ 1]$ , constant  $K=5$ , sliding surface coefficient matrix  $C^T=[4 \ 1]$  and first order filter time constant  $\tau = 0.01$ . Three cases of disturbance have been considered. These conditions are same for all three cases of disturbance.

Case I: The external disturbance considered is a unit step signal acting on the system at time  $t=1$ sec.

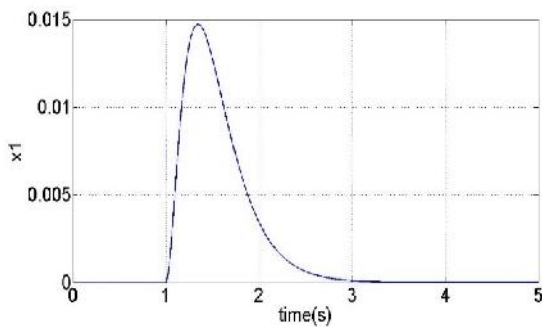


Fig. 1: State  $x_1$

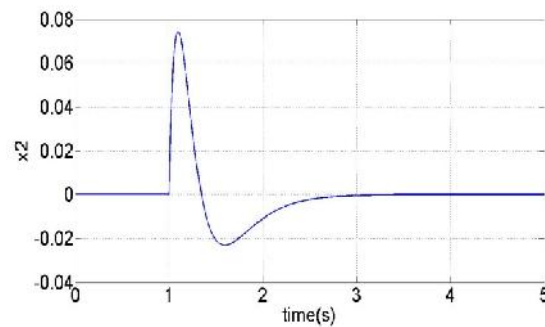


Fig. 2: State  $x_2$

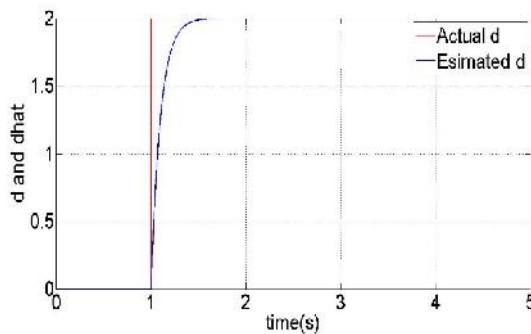


Fig. 3: Disturbance and its Estimate

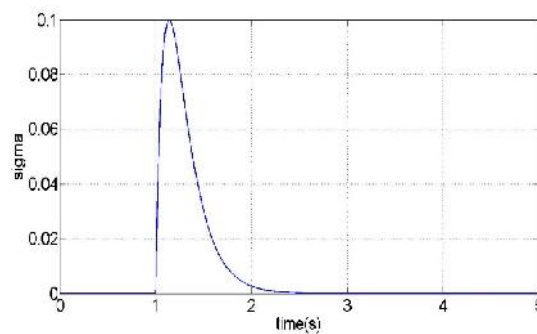


Fig. 4: Sigma

It can be seen from Fig. (1) and (2) that the state  $x_1$  and state  $x_2$  goes to zero irrespective of disturbance acting on the system. Fig. (3) shows the disturbance estimation capability of UDE.

Case II: The disturbance is sine wave of amplitude 1 and frequency 5 Hz.

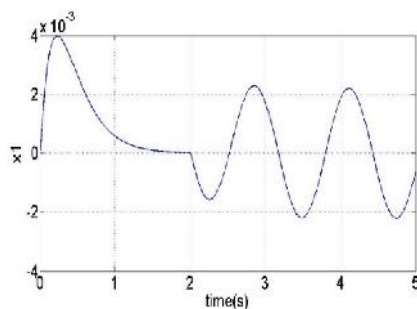


Fig. 5: State  $x_1$

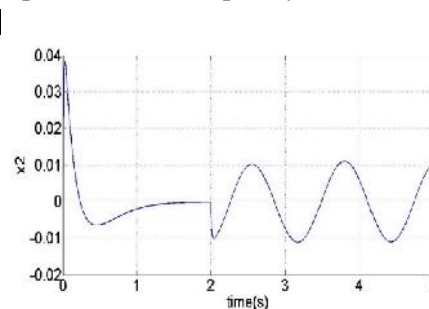


Fig. 6: State  $x_2$

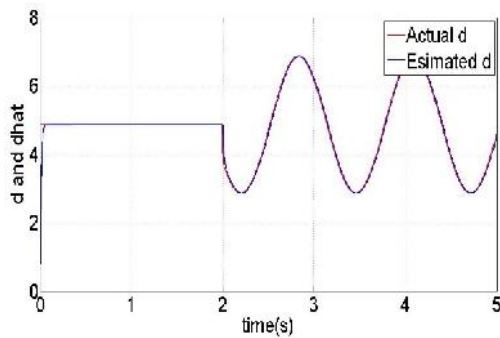


Fig. 7: Disturbance and its Estimate

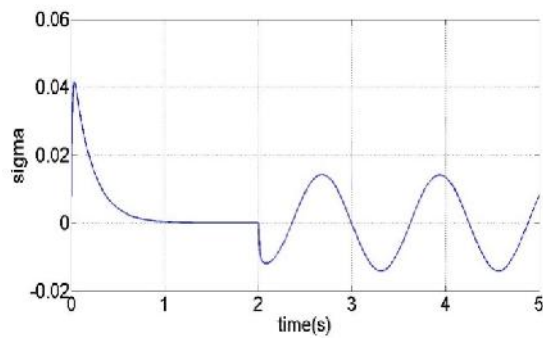


Fig. 8: Sigma

The system performance is tested with a sine wave disturbance of amplitude 1 and frequency 5 Hz. Sine wave disturbance is applied at 2 sec.

Case III: Complex disturbance is considered using signal builder.

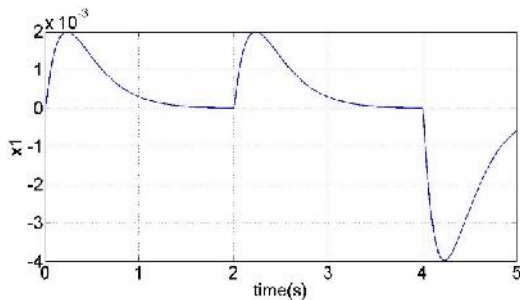


Fig. 9: State x1

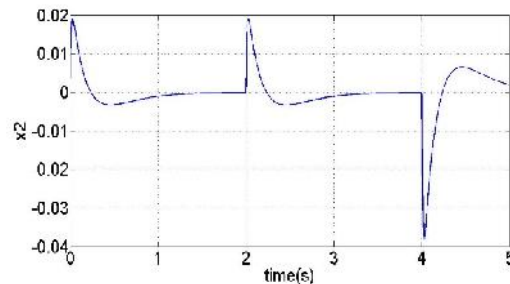


Fig. 10: State x1

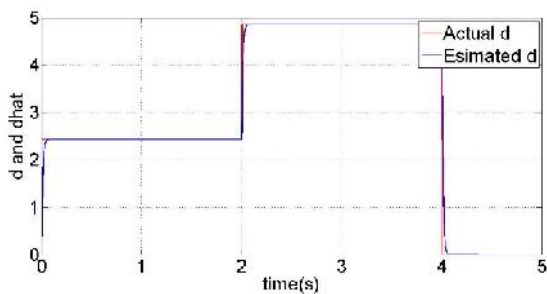


Fig. 11: Disturbance and its Estimate

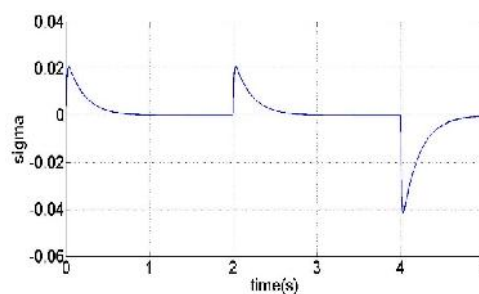


Fig. 12: Sigma

It can be seen from the Fig (9), (10) that the state  $x_1$  and the state  $x_2$  go to 0 inspite of disturbance action on it. The disturbance is estimated and shown in Fig. (11).

All the above simulations show that the UDE based control law is robust to different types of disturbances. However if the disturbances are fast varying then the order of the filter needs to be increased for getting satisfactory results.

## CONCLUSION

In this paper, the strategy of uncertainty and disturbance estimator (UDE) has been successfully employed against various types of disturbances. The control law thus derived has been validated using simulation on MATLAB.

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**REFERENCES**

- [1] Q.-C. Zhong and D. Rees, "Control of uncertain LTI systems based on an uncertainty and disturbance estimator," *ASME J. Dyn. Syst., Meas., Control*, vol. 126, pp. 905–910, 2004.
- [2] R. K. Stobart, A. Kuperman, and Q.-C. Zhong, "Uncertainty and disturbance estimator-based control for uncertain LTI-SISO systems with state delays," *ASME J. Dyn. Syst., Meas., Control*, vol. 133, no. 2, pp.1–6, Mar. 2011.
- [3] Youcef-Toumi, K., and Ito, O., "A Time Delay Controller for Systems With Unknown Dynamics," *ASME J. Dyn. Syst., Meas., Control*, pp. 133–142, 1990.
- [4] B. Ren, Q.-C. Zhong, and J. Chen, "Robust control for a class of non-affine nonlinear systems based on the uncertainty and disturbance estimator," *IEEE Trans. Ind. Electron.*, vol. 62, no. 9, pp. 5881–5888, Sept. 2015.
- [5] S. Talole and S. Phadke, "Robust input-output linearisation using uncertainty and disturbance estimation," *International Journal of Control*, vol. 82, no.10, pp. 1794–1803, 2010.
- [6] Alon Kuperman and Qing-Chang Zhong, "Robust UDE-Based Control of Uncertain Nonlinear State-Delay Systems", Dept. of Electrical Engineering and Electronics, University of Liverpool, UK, pp .928-933, 2009.
- [7] Jaywant P. Kolhe, Md Shaheed, T. S. Chandar and S. E. Talole, "Robust control of robot manipulators based on uncertainty and disturbance estimation", *International Journal of Robust and Nonlinear Control Int. J. Robust. Nonlinear Control*; 23: pp.104–122, 2013.
- [8] A. Kuperman, "UDE-based robust voltage control of DC-DC power converters", in 5th International Congress on Ultra Modern Telecommunications and Control Systems and Workshops (ICUMT), pp. 140–145, Sept. 2013.
- [9] Rohan Vimal Raj, Venkata Sreecharan Poluru, T S Chandar, "Robust control of Electric motor drive based on UDE", *Proceedings of the 9th IFAC symposium on Control of Power and Energy Systems*, Indian Institute of Technology Delhi, India, 2015.
- [10] Anish Ashok, Deepika Kumari, Kavya Satish, Swati Eswar and T. S. Chandar, "Robust Control of UAVs using Uncertainty and Disturbance Estimation", *International Conference on Industrial Instrumentation and Control (ICIC)* College of Engineering Pune, India, pp.434-438, 2015.
- [11] Jianjun Ren, Yongqiang Ye, Guofeng Xu, Qiangsong Zhao, Mingzhe Zhu, "Uncertainty and Disturbance Estimator-Based Current Control Scheme for PMSM Drives With a Simple Parameter Tuning Algorithm", *IEEE Transactions On Power Electronics* , accepted for publication, 2016
- [12] Akshata S Kori, C. M. Ananda and T. S. Chandar, "Robust Control of Single Axis Gimbal Platform for Micro Air Vehicles Based on Uncertainty and Disturbance Estimation", *7th International Conference on Mechanical and Aerospace Engineering*, pp.480-486, 2016.
- [13] Jinhao Chen, Beibei Ren and Qing-Chang Zhong, "UDE-Based Trajectory Tracking Control of Piezoelectric Stages", *IEEE Transactions On Industrial Electronics*, accepted for publication, 2016.
- [14] Alon Kuperman and Qing-Chang Zhong, "Robust UDE-Based Control of Uncertain Nonlinear State-Delay Systems", Dept. of Electrical Engineering and Electronics, University of Liverpool, UK, pp .928-933, 2009.