

Role of Diffusion, Doping and DC Magnetic Field on Parametric Amplification and Dispersion of Acoustic Phonons in Centrosymmetric Semiconductors

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ABSTRACT

Assuming that the origin of the nonlinear interaction lies in the second-order optical susceptibility $\chi^{(2)}$ arising from the diffusion-induced nonlinear current density, and using straight-forward coupled mode theory, parametric amplification/attenuation and dispersion of acoustic phonons is analytically investigated in a magnetized semiconductor plasma. The analysis deals with the qualitative behaviour of the threshold value of pump electric field E_{0th} for the onset of parametric amplification, parametric gain constant $g_a(\omega_a)$ and anomalous parametric dispersion $[\chi_d^{(2)}]_r$ with respect to excess doping concentration (via plasma frequency ω_p), applied magnetic field (via cyclotron frequency ω_c), and pump electric field E_0 for different values of diffusion coefficient D . The proper selection of ω_p , ω_c and E_0 enhances the positive and negative dispersion and gain profile, which may drastically reduce the fabrication cost of parametric devices based on this interaction.

Keywords

Parametric amplification/attenuation, parametric dispersion, acoustic phonon, centrosymmetric semiconductors, diffusion, magnetic field

INTRODUCTION

Parametric interaction (PI) of an intense laser beam (hereafter called 'pump') with nonlinear medium results into generation of waves at new frequencies through controlled splitting or mixing of the waves which may undergo amplification/attenuation depending on the properties of the medium and the geometry of applied field. Thus there are two distinct viewpoints of the said interaction: material properties and wave propagation characteristics. These phenomena can well be explained in terms of bunching of the free carriers present in the medium under the influence of the externally applied fields and those associated with the generated wave [1]. Thus any mechanism influencing the bunching of carriers is expected to modify the linear and nonlinear properties of the medium and hence the associated phenomena. Since bunching of carriers induces a density gradient in the medium, their diffusion becomes inevitable and thus can play a crucial role in parametric processes.

In a nonlinear medium the breakdown of the superposition principle may lead to interaction between waves of different frequencies. There exist a number of nonlinear interactions which can be classified as PI of coupled mode. In the phenomena of PI of coupled modes, the energy of external pump wave is transferred to the generated waves by a resonant mechanism that takes place when the field amplitude is large enough to cause the vibration (with the external field frequency) of certain physical parameters of the system. The phenomenon of PI plays a distinctive role in nonlinear optics. Parametric processes have been widely used to generate tunable coherent radiation in a nonlinear crystal that is not directly available from a laser source [2, 3]. Parametric amplifiers, parametric oscillators, optical phase conjugation, pulse narrowing, squeezed state generation, etc. are some of the important devices and processes whose origin lies in PI in a nonlinear medium. Besides these technological uses, there are several other applications of PI in which basic scientists are interested [4, 5]. It is well known fact that the origin of PI lies in the second order nonlinear optical susceptibility $\chi^{(2)}$ of the medium. Flytzanis [6] and Piepones [7] have, respectively, studied $\chi^{(2)}$ in different frequency regimes and the sum rules for the nonlinear susceptibilities in solids and other media. Until now a number of experiments have been performed concerning the behaviour of $\chi^{(2)}$; but nevertheless, the agreement between theory and experiments can be said to be poor.

Photo-induced light scattering in a nonlinear medium is an area of extensive research due to its manifold technological applications in optoelectronics. Acousto-optic (AO) interactions in dielectrics and semiconductors are playing an increasing role in optical modulation and beam steering [8, 9]. However, in integrated optoelectronic device applications, the AO modulation process becomes a serious limitation due to the high acoustic power requirements. The most direct approach to this problem is to tailor a new material with more desirable AO properties [10, 11]. An alternative method for amplifying the acoustic wave (AW) within the existing device is also being pursued [12-14]. The AW diffracts the light beam within the active medium and provides an effective mechanism for a nonlinear optical response in AO devices. Specifically, the photo-elastic effect in a medium causes a variation in the refractive index of medium which is proportional to an acoustic perturbation, and this implies the existence of a corresponding electrostrictive effect. It induces an acoustic response in the medium that is a spatially varying quadratic function of the local electric field.

The plasma effects in a semiconductor are a subject of continuous special attention, because of the attached technological interest and also for being a quite appropriate system for testing ideas and methods in the area of plasma physics. Although PI of waves has been extensively studied in the past few years [15-18], there are tremendous possibilities for further exploration and exploitation due to the poor agreement between theories and experiments [19-21]. The current trends in the field indicate that this old but fascinating phenomenon is still hotly pursued by both theoreticians as well as experimentalists, and an increasing number of interesting applications exploiting PI are being discovered or are yet to be discovered.

Recently, high mobility semiconductors have attracted much attention for their potential electronic and optical device applications. The high mobility of optically excited charge carriers makes diffusion effects particularly relevant in semiconductor technology as they (the charge carriers) travel significant distances before recombination. In most cases of investigations of nonlinear optical interactions particularly of parametric interactions, the nonlocal effects, such as diffusion of the excitation density that is responsible for the nonlinear refractive index change, have normally been ignored. The study of the reflection and transmission of a Gaussian beam incident upon an interface that separates a linear and nonlinear diffusive medium has stimulated an effort to include diffusion in the computation of nonlinear electromagnetic wave interactions in bulk and at nonlinear-nonlinear interfaces [22, 23].

Literature survey reveals that in most of the previous reported works, the diffusion effects of the excess carrier density and externally applied magnetic field has not been taken into the account. Hence, in the present paper we have presented an analytical investigation of second-order nonlinearity due to carrier diffusion current in an n-type centrosymmetric semiconductor plasma. The vanishingly small second-order nonlinearity can be enhanced in centrosymmetric media by creating favourable conditions through the adjustment of material parameters, wave propagation properties and externally applied magnetic field [24]. Interestingly, in the present work it has been shown that the diffusion of carriers may induce appreciable large second-order

nonlinearity in a magnetized diffusive semiconductor, which may be termed as ‘diffusion induced second order (DISO) nonlinearity’ and may lead to parametric dispersion and amplification of the AW in a collision dominated semiconductor plasma ($\nu \ll \omega$) due to a pump of frequency $\omega_0 \approx \nu$, ν being the momentum transfer collision frequency [25]. Moreover, this DISO nonlinearity is found to maximize at considerably low magnetic field ($\omega_c \approx 0.01\omega_0$; ω_c being cyclotron frequency). This interesting feature of DISO nonlinearity may play an extremely important role in operation of parametric amplifiers, oscillators, tunable radiation sources etc. Because the devices based on DISO nonlinearity will require very low magnetic field and thereby reducing their operating cost drastically.

The present investigation has been made under the following assumptions:

1. The crystal is assumed to be centrosymmetric so that the effects arising due to any pseudopotential have been neglected.
2. The pump electric field has been taken to be less than the damage threshold of the semiconductor crystal.
3. The semiconductor has an isotropic and nondegenerate conduction band.
4. The band nonparabolicity which contributes above 3% over the parabolic band structure has been neglected.
5. The ambient temperature of the crystal is assumed to be maintained at 77 K (because at low lattice temperature the dominant mechanisms for transfers of momentum and energy of electrons are assumed to be due to acoustic phonon and polar optical phonon scattering, respectively).
6. The crystal sample is irradiated by a laser with photon energy well below the band-gap energy of the semiconductor crystal, so that only free charge carriers influence the optical properties of the material.
7. The effects on optical properties of material due to photo-induced interband transition mechanisms are neglected throughout the analysis [25].

The analysis is based on the coupled mode theory for investigating the AW spectrum whose origin is due to the parametric three wave mixing process. The AO field couples with the internally generated signal in the presence of a strain, and attenuates/amplifies it under an appropriate phase matching condition.

This paper is organized in the following manner:

In section 2, the diffusion induced effective acousto-optical susceptibility describing the three wave interaction has been deduced from a single component fluid model and Maxwell’s equations. A linear stability analysis of the growth rates of the parametrically generated AW is presented. Section 3 includes the discussion of the analytical results obtained in the previous section. Section 4 lists the important conclusions that can be derived.

THEORETICAL FORMULATIONS

The phenomenon of PI of AW arises because of the coupling that the driving pump electric field introduces between the AW and the electron-plasma wave (EPW). In the multimode theory of PIs, an acoustic perturbation in the lattice gives rise to an electron density fluctuation in the medium at the same frequency. This couples nonlinearity with the pump field and drives the EPW at the sum and difference frequencies. This electron density perturbation, in turn, couples nonlinearity with external field and may reinforce the original perturbation at the acoustic frequency. Thus, under certain conditions, the AW and EPW derive each other unstable at the expense of the pump electric field.

In order to study the PI arising due to the three-wave interaction in an n-type diffusive semiconductor, an analytical expression for the DISO optical susceptibility $\chi_d^{(2)}$ for the AW has been derived in the medium.

The hydrodynamic model of n-type diffusive semiconductor plasma is considered. The suitability of this model seems without any loss of significant information, by replacing the streaming electrons with an electron fluid described by a few macroscopic parameters like average carrier density, average velocity etc. However, it restricts the analysis to be valid only in the limit $k_a l \ll 1$ (k_a the AW number, and l the carrier mean free

path). We consider a spatially uniform ($|k_0| \approx 0$) pump field $\vec{E} = \hat{x}E_0 \exp(i\omega_0 t)$ which irradiate an n-type diffusive semiconductor medium immersed in a transverse static magnetic field $\vec{B}_0 = \hat{z}B_0$. The PI of pump generates an AW at (ω_a, \vec{k}_a) and scatters a side band wave at (ω_1, \vec{k}_1) supported by the lattice and electron plasma in the medium, respectively. The momentum and energy exchange between these waves can be described by phase-matching conditions: $\hbar \vec{k}_0 \approx \hbar \vec{k}_1 + \hbar \vec{k}_a$ and $\hbar \omega_0 \approx \hbar \omega_1 + \hbar \omega_a$. In the interaction of high frequency electromagnetic waves and acoustic waves, it has been assumed without any loss of generality $|\vec{k}_a| (\approx k) \ll |\vec{k}_0|$ under the dipole approximation.

The basic equations describing PI of the pump with the medium are as follows:

$$\frac{\partial^2 u}{\partial t^2} - \frac{C}{\rho} \frac{\partial^2 u}{\partial x^2} + 2\Gamma_a \frac{\partial u}{\partial t} = \frac{1}{2\rho} \varepsilon(\eta^2 - 1) \frac{\partial}{\partial x} (\vec{E}_{eff} \cdot \vec{E}_1^*) \quad (1)$$

$$\frac{\partial \vec{v}_0}{\partial t} + \nu \vec{v}_0 = \frac{e}{m} \vec{E}_{eff} \quad (2)$$

$$\frac{\partial \vec{v}_1}{\partial t} + \nu \vec{v}_1 + \left(\vec{v}_0 \cdot \frac{\partial}{\partial x} \right) \vec{v}_1 = \frac{e}{m} \left(\vec{E}_1 + \vec{v}_1 \times \vec{B}_0 \right) \quad (3)$$

$$\frac{\partial n_1}{\partial t} + n_0 \frac{\partial v_1}{\partial x} + v_0 \frac{\partial n_1}{\partial x} + D \frac{\partial^2 n_1}{\partial x^2} = 0 \quad (4)$$

$$\vec{P}_{ao} = -\varepsilon(\eta^2 - 1) \nabla(u \cdot \vec{E}) \quad (5)$$

$$\frac{\partial \vec{E}_1}{\partial x} = \frac{n_1 e}{\varepsilon} + \frac{(\eta^2 - 1)}{\varepsilon_1} \vec{E}_{eff} \frac{\partial^2 u^*}{\partial x^2} \quad (6)$$

$$D = \frac{k_B T}{e} \mu. \quad (7)$$

The subscripts 0 and 1 refer to the physical quantities related to pump and side-band wave, respectively.

Equation (1) represents the motion of lattice vibrations in the crystal in which u , ρ , C , Γ_a and η are the relative displacement of oscillators from the mean position of the lattice, mass density, linear elastic modulus of the crystal, phenomenological acoustic damping parameter, and refractive index of the medium respectively. \vec{E}_{eff} represents the effective electric field, which includes the Lorentz force $\vec{v}_0 \times \vec{B}_0$ in the presence of an external magnetic field B_0 . The right hand side of Eq. (1) is an external driving force \vec{F}_u applied by the electromagnetic field.

Equations (2) and (3) represent the electron motion under the influence of the fields associated with the pump and side-band wave, respectively in which m and ν are the electron effective mass and phenomenological momentum transfer collision frequency of electrons respectively. Equation (4) is the continuity equation including diffusion effects in which n_0 , n_1 and D are the equilibrium and perturbed electron densities and diffusion coefficient respectively. In an AO medium an acoustic mode is generated due to the electrostrictive strain leading to the energy exchange between the electromagnetic and acoustic fields. Under the influence of the electromagnetic field, the ions within the lattice move in a non-centrosymmetrical position usually producing a contraction in the direction of the field and an expansion across it. The electrostatic force thus produced is the origin of the AO strain within the medium. Equation (5) describes that the AW generated due to the electrostrictive strain modulates the dielectric constant and gives rise to a nonlinear induced polarization

P_{ao} . This polarization results in the coupling of the space-charge wave with traveling acoustic grating. The magnitude of space charge field thus depends on the refractive index grating strength that is proportional to the generated acoustic field strength. The space charge field therefore couples the pump and the signal field in the presence of the acoustic grating. The space charge field E_1 is determined from Poisson equation (6) where, ϵ_1 is the dielectric constant of the crystal. The diffusion coefficient D is determined from Einstein's relation (7) in which, k , T and μ are Boltzmann constant, electron temperature, and electron mobility respectively. The induced current density $J(x, t)$ in the present case is assumed to consist of a diffusion term only near thermal equilibrium at temperature T so that the analysis shall be confined only to the role of diffusion current on PI. The AW and side-band wave perturbations are assumed to vary as $\exp[i(k_{a,1}x - \omega_{a,1}t)]$.

The induced charge carriers are subjected to diffusion in the spatially varying intensity of the interfering beams, while the electric field associated with the resultant space charge operates through the AO effect to modify the refractive index of the medium. The three waves parametric coupling of the optical and acoustic (material density) waves in the semiconductor are assumed to occur due to bunching of the carriers produced by the fields associated with the various waves generated in the crystal as a result of the nonlinear mixing of the fundamental waves itself. Thus, any process such as diffusion, enhancing carrier bunching will lead to an amplification of the parametrically generated output signal.

The interaction of the pump with parametrically generated AW produces an electron density perturbation which, in turn, derives an EPW in the medium. Thus by using the standard approach [18], the equation of the EPW is obtained from Eqs. (1) – (7) as:

$$\frac{\partial^2 n_1}{\partial t^2} + v \frac{\partial n_1}{\partial t} + \bar{\omega}_p^2 n_1 + vD \frac{\partial^2 n_1}{\partial x^2} - \frac{n_0 e k_a^2}{m \epsilon_1} E_{eff} u^* = -\frac{e}{m} \frac{\partial n_1}{\partial x} E_{eff}, \quad (8)$$

$$\text{where } \bar{\omega}_p^2 = \omega_p^2 \left(1 + \frac{\omega_c^2}{v^2} \right)^{-1}$$

Here $\omega_p^2 = \frac{n_0 e^2}{m \epsilon}$ is the plasma frequency and $\omega_c = \frac{e B_0}{m}$ is the cyclotron frequency of the carriers.

In deriving Eq. (8), the Doppler shift due to traveling space charge wave is neglected under the assumption $\omega_0 \gg k_0 v_0$. This equation describes coupling between the AW and side-band wave in the presence of an intense pump. The energy flow from the pump to the generated waves shall be maximum when the process is phase matched. Resolving Eq. (8) and using the rotating wave approximation (RWA), the slow component (n_s) associated with the AW that produces the density perturbation at frequency ω_a and the fast component (n_f) associated with side-band wave that produces the perturbation at frequency $\omega_1 \approx (\omega_0 \pm p\omega_a)$, p being an integer; are obtained.

By neglecting the off-resonant frequencies $p \geq 2$ [26], we get

$$\frac{\partial^2 n_f}{\partial t^2} + v \frac{\partial n_f}{\partial t} + \bar{\omega}_p^2 n_f + vD \frac{\partial^2 n_f}{\partial x^2} - \frac{n_0 e k^2}{m \epsilon_1} (\eta^2 - 1) E_{eff} u^* = -\frac{e}{m} \frac{\partial n_s^*}{\partial x} E_{eff} \quad (9a)$$

and

$$\frac{\partial^2 n_s}{\partial t^2} + v \frac{\partial n_s}{\partial t} + \bar{\omega}_p^2 n_s + vD \frac{\partial^2 n_s}{\partial x^2} = -\frac{e}{m} \frac{\partial n_f}{\partial x} E_{eff}. \quad (9b)$$

Eqs. (9) reveal that the slow and fast components of the electron density perturbations are coupled to each other via the pump electric field. Hence the presence of a pump field is the fundamental necessity for PI to occur.

The usage of Eqs. (1) and (9a,b) and mathematical simplification allow us to calculate the low- and high-frequency components of the density perturbations as:

$$n_f = \frac{n_0 e k^2 (\eta^2 - 1) \Phi E_{eff}}{m \epsilon_1} \quad (10a)$$

and

$$n_s = - \frac{\epsilon_0 n_0 k^2 \omega_0^2 (\eta^2 - 1)^2 \Phi E_0 E_1^*}{2 \rho (\omega_a^2 - k_a^2 v_a^2 + 2i \Gamma_a \omega_a) (\omega_0^2 - \omega_c^2 + 2iv \omega_0)} \quad (10b)$$

$$\text{where } \Phi = \left(1 - \frac{\delta_1^2 \delta_2^2}{k^2 (e/m)^2 E_{eff}^2} \right)^{-1}, \quad \delta_1^2 = \bar{\omega}_p^2 - \omega_1^2 - v D k^2 \quad \text{and} \quad \delta_a^2 = \bar{\omega}_p^2 - \omega_a^2 - v D k^2.$$

The term $(\omega_a^2 - k_a^2 v_a^2 + 2i \Gamma_a \omega_a)$ represents AW dispersion in the presence of damping, the quantity in the square bracket represents the dispersion of pump wave due to collision and diffusion of charge carriers and $n_0 e (\eta^2 - 1)$ is the AO coupling parameter in an electrostrictive medium

In order to study the role of diffusion on the nonlinearity of the medium, we express the diffusion-induced current density at the acoustic frequency by the relation:

$$J_d(\omega_a) = e D \frac{\partial n_s}{\partial x}. \quad (11)$$

In the coupled-mode approach, the time integral of nonlinear current density $J_d(\omega_a)$ yields the nonlinear-induced polarization

$$P_d(\omega_a) = \int J_d(\omega_a) dt = \frac{\epsilon_0 n_0 e D k^3 \omega_0^2 (\eta^2 - 1)^2 \Phi E_0 E_1^*}{2 \rho \omega_a (\omega_a^2 - k_a^2 v_a^2 + 2i \Gamma_a \omega_a) (\omega_0^2 - \omega_c^2 + 2iv \omega_0)}. \quad (12)$$

The DISO susceptibility $\chi_d^{(2)}$ can be obtained by defining the nonlinear polarization as:

$$P_d(\omega_a) = \epsilon_0 \chi_d^{(2)} E_0 E_1^*, \quad (13)$$

which gives

$$\chi_d^{(2)} = \frac{\epsilon_0 n_0 e D k^3 \omega_0^2 (\eta^2 - 1)^2 \Phi}{2 \rho \omega_a (\omega_a^2 - k_a^2 v_a^2 + 2i \Gamma_a \omega_a) (\omega_0^2 - \omega_c^2 + 2iv \omega_0)}. \quad (14)$$

The above equation reveals that diffusion of the carriers induces second-order nonlinearity in the medium which would otherwise be absent or vanishingly small in a centrosymmetric medium.

Now rationalizing Eq. (14), we obtain the real $[\chi_d^{(2)}]_r$ and imaginary $[\chi_d^{(2)}]_i$ parts of the complex DISO susceptibility $\chi_d^{(2)}$ using the relation $\chi_d^{(2)} = [\chi_d^{(2)}]_r + i [\chi_d^{(2)}]_i$:

$$[\chi_d^{(2)}]_r = \frac{n_0 e D k^3 \omega_0^2 (\eta^2 - 1)^2 \Phi [(\omega_a^2 - k_a^2 v_a^2)(\omega_0^2 - \omega_c^2) - 4v \Gamma_a \omega_a \omega_0]}{2 \rho \omega_a [(\omega_a^2 - k_a^2 v_a^2)^2 + 4 \Gamma_a^2 \omega_a^2][(\omega_0^2 - \omega_c^2)^2 + 4v^2 \omega_0^2]} \quad (15a)$$

and

$$[\chi_d^{(2)}]_i = \frac{-n_0 e D k^3 (\eta^2 - 1)^2 \Phi [\Gamma_a \omega_a (\omega_0^2 - \omega_c^2) + v \omega_0 (\omega_a^2 - k_a^2 v_a^2)]}{2 \rho \omega_a [(\omega_a^2 - k_a^2 v_a^2)^2 + 4 \Gamma_a^2 \omega_a^2][(\omega_0^2 - \omega_c^2)^2 + 4v^2 \omega_0^2]}. \quad (15b)$$

The above formulation reveals that the crystal susceptibility is influenced by the carrier concentration n_0 (via ω_p) and by the transverse dc magnetic field B_0 (via ω_c) Eqs. (15a) and (15b) can be respectively employed to study the dispersion and amplification/attenuation characteristics of the scattered waves in the parametric process.

As is well-known, parametric amplification can be achieved at excitation intensities above a certain threshold value. This threshold nature can be obtained by setting $[\chi_d^{(2)}]_i = 0$ as:

$$[E_{0th}]_{para} = \frac{\delta_1 \delta_2}{(e/m)k}. \quad (16)$$

The amplification of the co-propagating waves in the electrostrictive medium is due to the linear dispersion effects in combination with the nonlinear processes. The steady state gain coefficient ($g_a(\omega_a)$) of a parametrically excited wave-form of the pump field exceeding a threshold value is obtained through the relation [27]:

$$g_a(\omega_a) = -\frac{k}{2\varepsilon_1} [\chi_d^{(2)}]_i E_0 = \frac{n_0 e D k^4 (\eta^2 - 1)^2 \Phi[\Gamma_a \omega_a (\omega_0^2 - \omega_c^2) + v \omega_0 (\omega_a^2 - k_a^2 v_a^2)] E_0}{2\varepsilon_1 \rho \omega_a [(\omega_a^2 - k_a^2 v_a^2)^2 + 4\Gamma_a^2 \omega_a^2][(\omega_0^2 - \omega_c^2)^2 + 4v^2 \omega_0^2]}. \quad (17)$$

The nonlinear parametric gain of the AW can be possible only if $[\chi_d^{(2)}]_i$ obtained from Eq. (15b) is negative, which is expected at pump electric field $|E_0| > |[E_{0th}]_{para}|$.

RESULTS AND DISCUSSION

The study presented in the preceding section clearly reveals that we may observe second-order nonlinearity in a weakly magnetized diffusive semiconductor. As a typical case, the numerical estimation has been made for n-type III-V semiconductor crystal at 77 K duly irradiated by a nanosecond pulsed 10.6 μm CO₂ laser. The physical constants used are [18]: $\rho = 5.8 \times 10^3 \text{ kg m}^{-3}$, $m = 0.0145 m_e$ (m_e the free mass of electron), $v = 3 \times 10^{11} \text{ s}^{-1}$, $\varepsilon_1 = 15.8$, $\omega_a = 10^{12} \text{ s}^{-1}$, $v_a = 4 \times 10^3 \text{ ms}^{-1}$, $\omega_0 = 1.78 \times 10^{14} \text{ s}^{-1}$, $\Gamma_a = 2 \times 10^{10} \text{ s}^{-1}$, $\eta = 3.9$.

Threshold Characteristics

The parametric growth of generated AW requires pump electric field to exceed certain threshold value E_{0th} , whose dependence on different parameters such as k , ω_p , ω_c etc. may be studied from Eq. (16). It is clear from this equation that an increase in the value of $\omega_c(B_0)$ will decrease the value of δ_1 and δ_2 and hence E_{0th} . As an illustration, Fig. 1 shows variation of E_{0th} as a function of AW number k for $n_0 = 10^{24} \text{ m}^{-3}$ and $\omega_c = 0.01\omega_0$. Curves (a), (b) and (c) represent the features for $D = 0.2, 0.3$ and $0.5 \text{ m}^2 \text{ s}^{-1}$ respectively. It can be seen that in all the three cases E_{0th} decreases sharply with increase in k acquiring a minimum value $(E_{0th})_{\min} = 8.2 \times 10^5, 6 \times 10^5$ and $2.7 \times 10^5 \text{ V m}^{-1}$ at $k = 8.5 \times 10^7, 6.7 \times 10^7$ and $3.6 \times 10^7 \text{ m}^{-1}$ for $D = 0.2, 0.3$ and $0.5 \text{ m}^2 \text{ s}^{-1}$ respectively. The increase in value of D decreases the value of $(E_{0th})_{\min}$ and shifts towards a lower value of k .

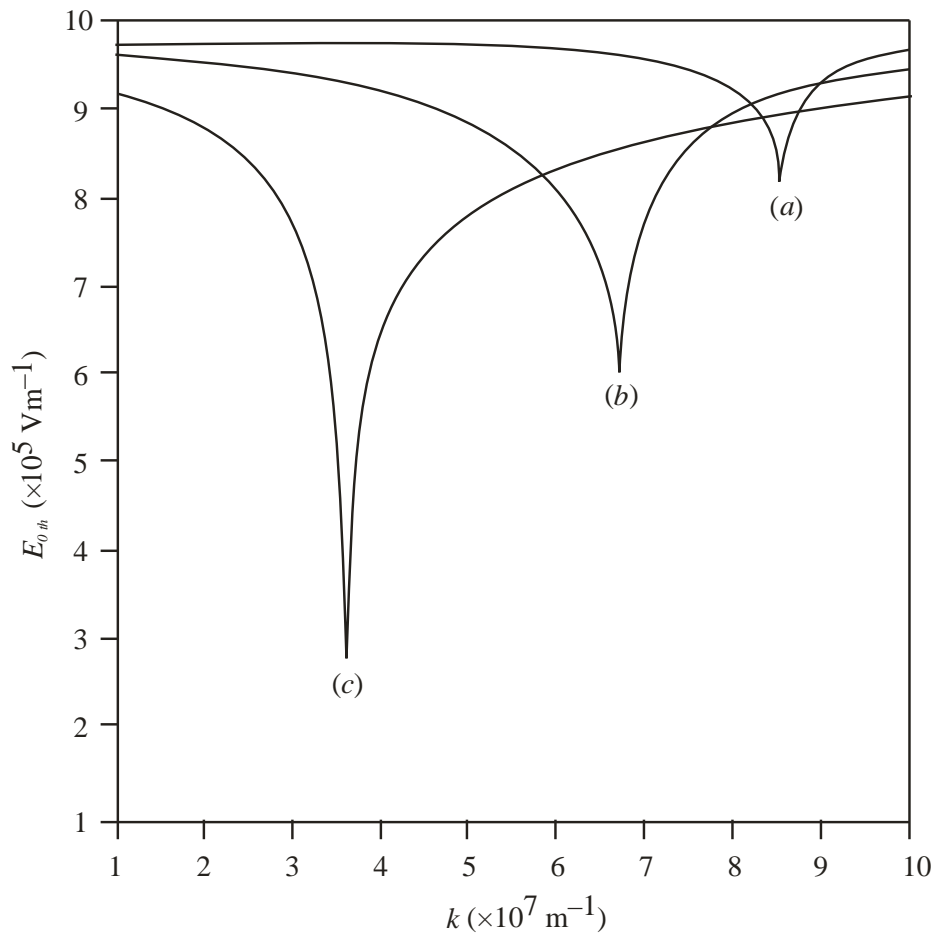


Fig. 1. Variation of threshold pump amplitude E_{0th} on AW number k for $n_0 = 10^{24}\text{m}^{-3}$, $\epsilon_c = 0.01$.
Curves (a), (b) and (c) are for $D = 0.2, 0.3$ and $0.5 \text{ m}^2\text{s}^{-1}$ respectively.

Using Eq. (16), it can be shown that at a particular value of ω_c the dip of E_{0th} corresponds to

$$k = \omega_p^2 \left(\frac{\omega_l}{vD} \right)^{1/2} \left(1 + \frac{\omega_c^2}{v^2} \right)^{-1} = k_m$$

and thus influenced by the carrier concentration n_0 (via ω_p), the magnetic field B_0 (via ω_c), and diffusion coefficient D . Obviously, in a heavily doped sample, E_{0th} minimizes at particular value of k while an increase in applied magnetic field and/or diffusion of the carriers further minimizes $(E_{0th})_{\min}$ and shifts towards lower values of k .

Parametric Amplification

The quantitative analysis of the parametric gain constant of AW $g_a(\omega_a)$ associated with parametric excitation process in n-type III-V diffusive semiconductor as a function of different parameters such as E_0 , ω_p , ω_c etc. may be studied from Eq. (17).

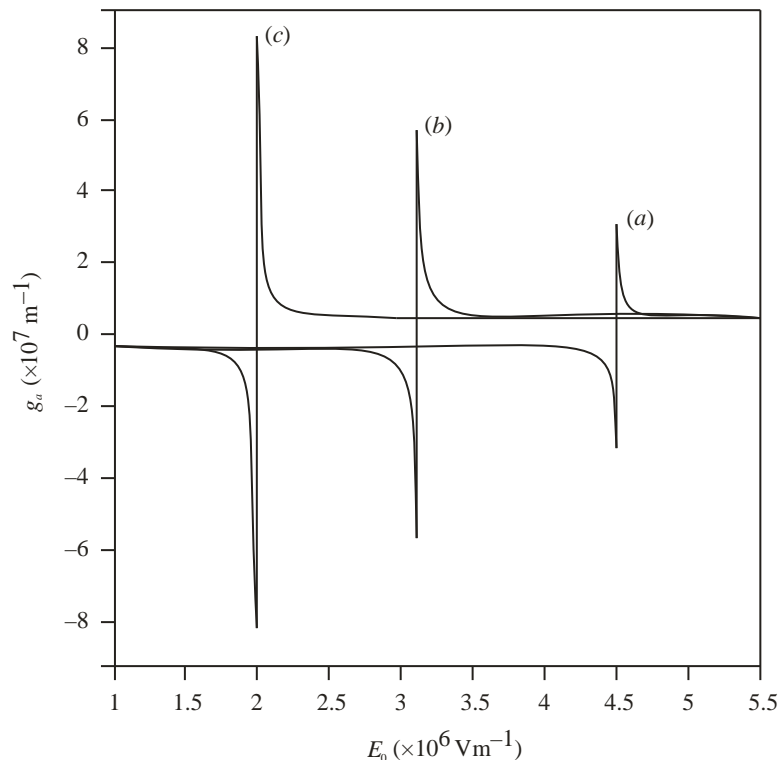


Fig. 2. Variation of gain constant of AW g_a with pump field E_0 for $n_0 = 10^{24}\text{m}^{-3}$, $\epsilon_c = 0.01$, $k = 5 \times 10^7 \text{m}^{-1}$. Curves (a), (b) and (c) are for $D = 0.2, 0.3$ and $0.5 \text{m}^2\text{s}^{-1}$ respectively.

Fig. 2 displays the variation of gain constant of AW $g_a(\omega_a)$ with pump field E_0 for $n_0 = 10^{24}\text{m}^{-3}$, $\omega_c = 0.01\omega_0$, $k = 5 \times 10^7 \text{m}^{-1}$. Curves (a), (b) and (c) represent the features for $D = 0.2, 0.3$ and $0.5 \text{m}^2\text{s}^{-1}$ respectively. For these set of data, the values of E_{0th} is found to be equal to $4.5 \times 10^6, 3.2 \times 10^6$ and $2 \times 10^6 \text{Vm}^{-1}$ respectively. It may be inferred that in all the three cases, for $E_0 < E_{0th}$, $g_a(\omega_a)$ is negative (which indicates absorption) and remains almost constant with the increase in E_0 . However, as E_0 approaches E_{0th} , $g_a(\omega_a)$ falls rapidly acquiring minimum value ($-3.2 \times 10^7, -5.8 \times 10^7$ and $-8.2 \times 10^7 \text{m}^{-1}$ for $D = 0.2, 0.3$ and $0.5 \text{m}^2\text{s}^{-1}$ respectively) followed by a very sharp rise making $g_a(\omega_a) = 0$ at $E_0 = E_{0th}$. Beyond this point, $g_a(\omega_a)$ becomes positive (which indicates amplification) and shoots up to its maximum value ($3.2 \times 10^7, 5.8 \times 10^7$ and $8.2 \times 10^7 \text{m}^{-1}$ for $D = 0.2, 0.3$ and $0.5 \text{m}^2\text{s}^{-1}$ respectively) beyond which the gain constant starts decreasing and becomes minimum. While comparing the results of curves (a), (b) and (c), it can be observed that with increasing D , $g_a(\omega_a)$ increases and the corresponding E_{0th} shifts towards lower values. A rise in gain constant is again witnessed if E_0 is further increased which may be explained as follows: The gain increases with E_0 overcoming the attenuation below threshold field and exhibit an abrupt rise due to modification of the effective second-order susceptibility which is induced by the space charge field. In the AO device, the AO interaction parameter is modulated by the diffusion current and the modified AW frequency ($\omega_a^2 - vDk^2$) under the influence of the intense induced pump field E_0 . Beyond this point, the intensity of space charge wave is increased resulting in a reverse transfer of energy from the AO field to the material waves in the resonant regime resulting in fall of gain. However as the pump field increases

further, it becomes strong enough to derive the space-charge waves overcoming dragging effect due to frictional forces and thereby again exhibit rise in gain. Hence this variation pattern may be attributed to factor Φ in Eq. (17). Thus by suitably choosing the strength of pump field, we may control the amplification/attenuation characteristics of the medium for the generated AW.

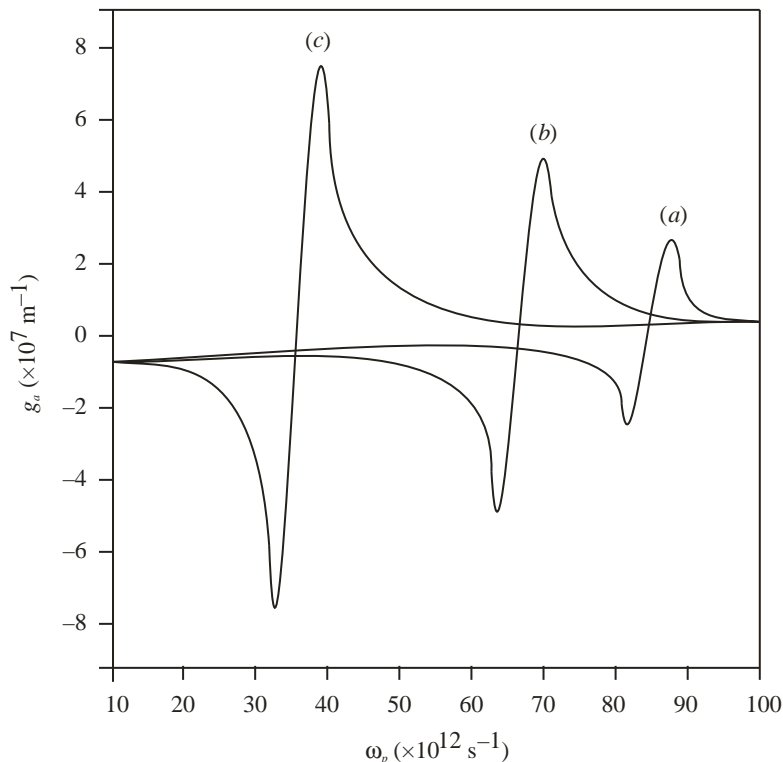


Fig. 3. Variation of gain constant of AW g_a with carrier concentration (via ω_p) for $\omega_c = 0.01 \omega_0$, $k = 5 \times 10^7 \text{ m}^{-1}$, $E_0 = 5 \times 10^6 \text{ Vm}^{-1}$. Curves (a), (b) and (c) are for $D = 0.2, 0.3$ and $0.5 \text{ m}^2\text{s}^{-1}$ respectively.

Fig. 3 shows the variation of $g_a(\omega_a)$ with carrier concentration n_0 (via ω_p) for $\omega_c = 0.01\omega_0$, $k = 5 \times 10^7 \text{ m}^{-1}$, $E_0 = 5 \times 10^6 \text{ Vm}^{-1}$. Curves (a), (b) and (c) represent the features for $D = 0.2, 0.3$ and $0.5 \text{ m}^2\text{s}^{-1}$ respectively. It is evident from Eq. (16) that E_{0th} reduces appreciably with an increase in carrier concentration (via parameters δ_1 and δ_2) and hence strongly depends upon it. It may be observed from Fig. 3 that at lower carrier concentration where the considered value of E_0 lies below the threshold value, the gain constant is negative and remains almost constant with increase in ω_p . However, as ω_p reaches the value for which E_0 becomes the threshold field, $g_a(\omega_a)$ falls rapidly acquiring a minimum value ($-2.4 \times 10^7, -4.8 \times 10^7$ and $-7.7 \times 10^7 \text{ m}^{-1}$ for $D = 0.2, 0.3$ and $0.5 \text{ m}^2\text{s}^{-1}$ respectively) followed by a sharp rise making $g_a(\omega_a) = 0$ (at $\omega_p = 8.4 \times 10^{13}, 6.7 \times 10^{13}$ and $3.6 \times 10^{13} \text{ s}^{-1}$ for $D = 0.2, 0.3$ and $0.5 \text{ m}^2\text{s}^{-1}$ respectively). Beyond this point, $g_a(\omega_a)$ becomes positive and shoots up to its maximum value ($2.4 \times 10^7, 4.8 \times 10^7$ and $7.7 \times 10^7 \text{ m}^{-1}$ for $D = 0.2, 0.3$ and $0.5 \text{ m}^2\text{s}^{-1}$ respectively) beyond which the gain constant starts decreasing and saturates to a very small value. While comparing the results of curves (a), (b)

and (c), it can be observed that increase in value of D decreases/increases peak value of $g_a(\omega_a)$ and shifts towards lower values of ω_p .

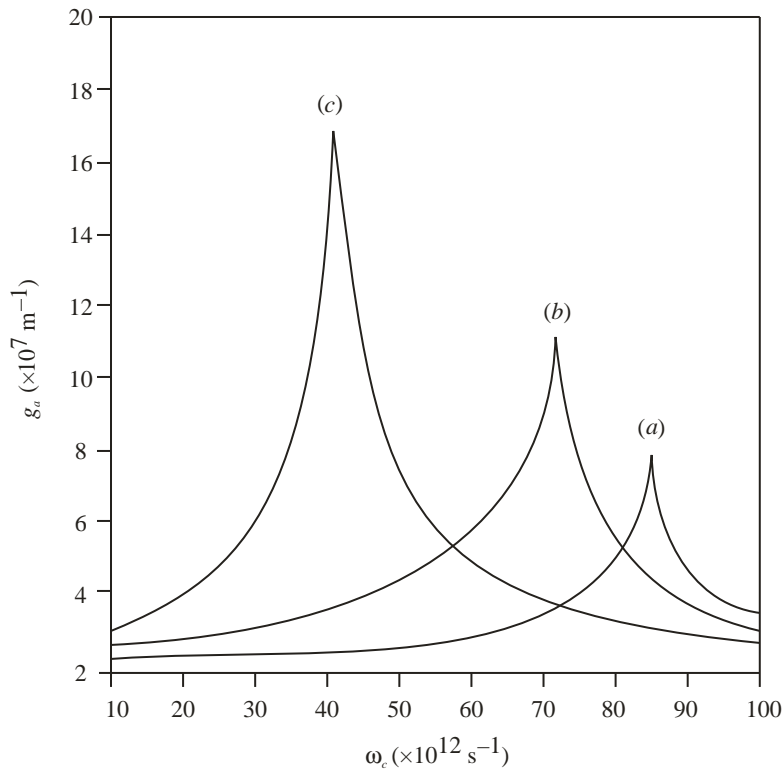


Fig. 4. Variation of gain constant of AW g_a with magnetic field (via ω_c) for $n_0 = 10^{24} \text{ m}^{-3}$, $k = 5 \times 10^7 \text{ m}^{-1}$, $E_0 = 5 \times 10^6 \text{ Vm}^{-1}$. Curves (a), (b) and (c) are for $D = 0.2, 0.3$ and $0.5 \text{ m}^2 \text{ s}^{-1}$ respectively.

Fig. 4 shows the variation of $g_a(\omega_a)$ with magnetic field B_0 (via ω_c) for $n_0 = 10^{24} \text{ m}^{-3}$, $k = 5 \times 10^7 \text{ m}^{-1}$, $E_0 = 5 \times 10^6 \text{ Vm}^{-1}$. Curves (a), (b) and (c) represent the features for $D = 0.2, 0.3$ and $0.5 \text{ m}^2 \text{ s}^{-1}$ respectively. At $\omega_c \ll \omega_0$, $g_a(\omega_a)$ increases sharply due to increase in parameter Φ and attains a maximum value ($8 \times 10^7, 1.1 \times 10^8$ and $1.7 \times 10^8 \text{ m}^{-1}$ at $\omega_c = 8.5 \times 10^{13}, 7.2 \times 10^{13}$ and $4.1 \times 10^{13} \text{ s}^{-1}$ for $D = 0.2, 0.3$ and $0.5 \text{ m}^2 \text{ s}^{-1}$ respectively). A further increase in ω_c (when $\omega_c \gg \nu$) causes a rapid decrease in parameter Φ and hence the growth rate of $g_a(\omega_a)$ of the AW. While comparing the results of curves (a), (b) and (c), it can be observed that increase in value of D increases $g_a(\omega_a)$ and shifts towards lower values of ω_c .

Parametric Dispersion

Being one of the principal objectives of the present analysis, the nature of parametric dispersion arising due to the real part of the second-order optical susceptibility, viz., $[\chi_d^{(2)}]_r$ as a function of different parameters such as ω_p , ω_c etc. can be studied from Eq. (15a).

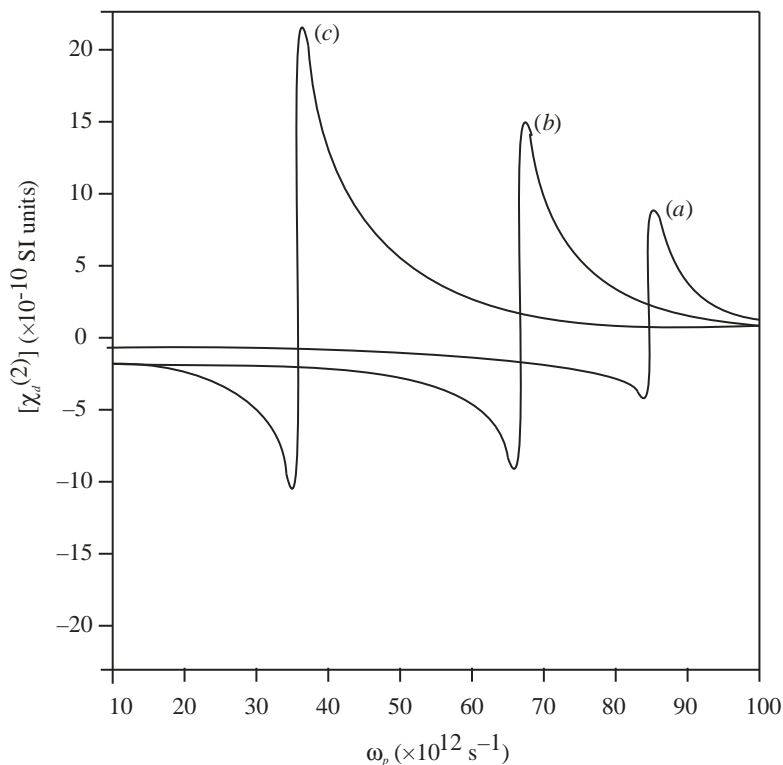


Fig. 5. Variation of parametric dispersion $\left[\chi_d^{(2)}\right]_r$ with carrier concentration (via ω_p) for $\omega_c = 0.01 \omega_0$, $k = 5 \times 10^7 \text{ m}^{-1}$, $E_0 = 5 \times 10^6 \text{ Vm}^{-1}$. Curves (a), (b) and (c) are for $D = 0.2, 0.3$ and $0.5 \text{ m}^2 \text{ s}^{-1}$ respectively.

Fig. 5. Shows the variation of parametric dispersion $\left[\chi_d^{(2)}\right]_r$ with carrier concentration (via ω_p) for $\omega_c = 0.01 \omega_0$, $k = 5 \times 10^7 \text{ m}^{-1}$, $E_0 = 5 \times 10^6 \text{ Vm}^{-1}$. Curves (a), (b) and (c) represent the features for $D = 0.2, 0.3$ and $0.5 \text{ m}^2 \text{ s}^{-1}$ respectively. It can be seen that there exists a distinct anomalous parametric dispersion regime that varies in magnitude with diffusion coefficient D . It appears worth mentioning that $\left[\chi_d^{(2)}\right]_r$ can be both positive and negative under the anomalous regime. At lower carrier concentration where the considered value of pump amplitude $E_0 < E_{0th}$, $\left[\chi_d^{(2)}\right]_r$ is negative and remains almost constant with increase in ω_p . However, as ω_p reaches the value for which $E_0 = E_{0th}$, $\left[\chi_d^{(2)}\right]_r$ falls rapidly acquiring a minimum value (-4×10^{-10} , -9×10^{-10} and -11×10^{-10} SI units for $D = 0.2, 0.3$ and $0.5 \text{ m}^2 \text{ s}^{-1}$ respectively) followed by a very sharp rise making $\left[\chi_d^{(2)}\right]_r = 0$ (at $\omega_p = 8.4 \times 10^{13}$, 6.7×10^{13} and $3.6 \times 10^{13} \text{ s}^{-1}$ for $D = 0.2, 0.3$ and $0.5 \text{ m}^2 \text{ s}^{-1}$ respectively). Beyond this point, $\left[\chi_d^{(2)}\right]_r$ becomes positive and shoots up to its maximum value (9×10^{-10} , 15×10^{-10} and 21×10^{-10} SI units for $D = 0.2, 0.3$ and $0.5 \text{ m}^2 \text{ s}^{-1}$ respectively) beyond which the $\left[\chi_d^{(2)}\right]_r$ starts decreasing and saturates to a very small value. The increase in value of diffusion coefficient D enhances the value of $\left[\chi_d^{(2)}\right]_r$ and shifts the anomalous dispersion regime towards lower values of ω_p . Such

type of variation of $\left[\chi_d^{(2)}\right]_r$ with doping concentration in III-V semiconductors was reported by Singh et.al. [18].

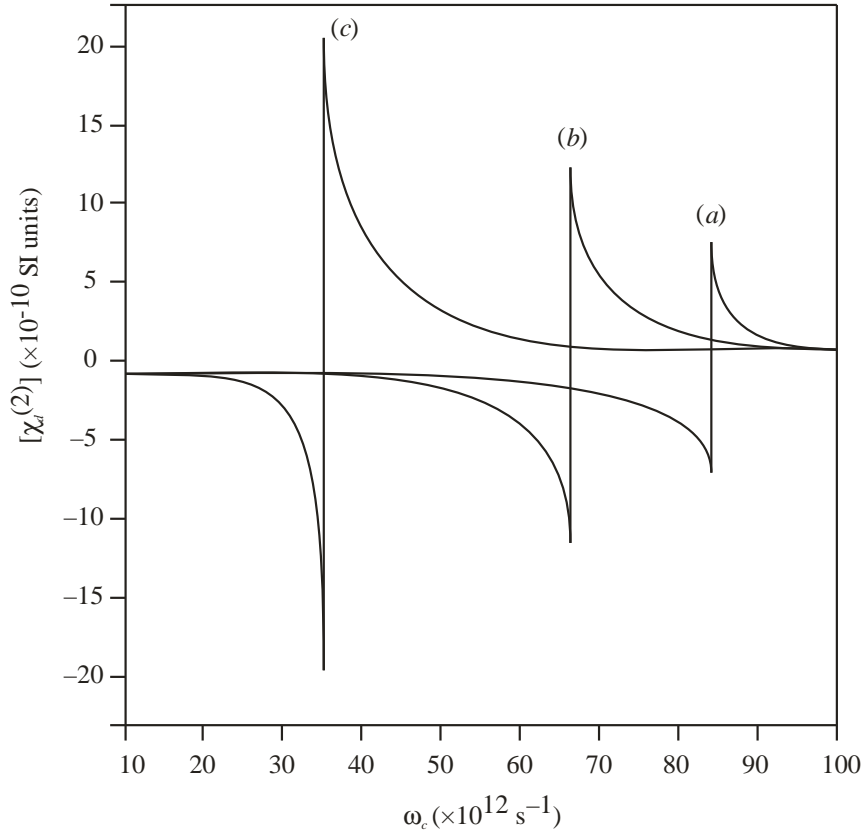


Fig. 6. Variation of parametric dispersion $\left[\chi_d^{(2)}\right]_r$ with magnetic field (via ω_c) for $n_0 = 10^{24} \text{ m}^{-3}$, $k = 5 \times 10^7 \text{ m}^{-1}$, $E_0 = 5 \times 10^6 \text{ Vm}^{-1}$. Curves (a), (b) and (c) are for $D = 0.2, 0.3$ and $0.5 \text{ m}^2 \text{ s}^{-1}$ respectively.

Fig. 6 shows the variation of parametric dispersion $\left[\chi_d^{(2)}\right]_r$ with magnetic field (via ω_c) for $n_0 = 10^{24} \text{ m}^{-3}$, $k = 5 \times 10^7 \text{ m}^{-1}$, $E_0 = 5 \times 10^6 \text{ Vm}^{-1}$. Curves (a), (b) and (c) represent the features for $D = 0.2, 0.3$ and $0.5 \text{ m}^2 \text{ s}^{-1}$ respectively. It may be inferred from this figure that for a particular value of D , with increase in ω_c , $\left[\chi_d^{(2)}\right]_r$ decreases and attains minimum value. A slight tuning in the value of ω_c beyond this point causes sharp increase in $\left[\chi_d^{(2)}\right]_r$ and attains a maximum value. With further increase in value of ω_c , $\left[\chi_d^{(2)}\right]_r$ decreases rapidly and saturates at larger values of doping concentration. The increase in value of diffusion coefficient D enhances the value of $\left[\chi_d^{(2)}\right]_r$ and shifts the anomalous dispersion regime towards lower values of ω_c .

The present theory thereby provides an insight into developing potentially useful diffusion-induced acousto-optical parametric amplifiers by incorporating the material characteristics of the medium.

REFERENCES

- [1] Conwell, E.M., Ganguly, A.K., 1971 "Mixing of acoustic waves in piezoelectric semiconductors", *Phys. Rev. B* 4, 2535-2558.
- [2] Krochik, G.M., 1975 "Conversion of radio frequency in four-wave parametric resonance processes based on stimulated Raman scattering", *Sov. J. Quantum Electron.* 5, 917-924.
- [3] Ding, Yj, Khurgin, J.B., 1998 "Generation of tunable coherent far-infrared waves based on backward optical parametric oscillation in gallium selenide", *J. Opt. Soc. Am. B* 15, 1567-1571.
- [4] Pecchia, A., Laurito, M., Apai, P., Danailov, M.B., 1999 "Studies of two wave mixing of very broad spectrum laser light in BaTiO₃", *J. Opt. Soc. Am. B* 16, 917-923.
- [5] Tokman, I.D., Vugalter, G.A., Grebeneva, A.I., 2005 "Parametric interaction of two acoustic waves in a crystal of molecular magnets in the presence of a strong ac magnetic field", *Phys. Rev. B* 71, 094431.
- [6] Flytzanis, Ch., 1975 "Quantum Electronics", Academic Press, New York, p. 9.
- [7] Piepones, K.E., 1987 "Sum rules for nonlinear susceptibilities in case of difference frequency generation", *J. Phys. C: Solid State Phys.* 20, 285-292.
- [8] Tervonen, E., Friberg, A.T., Turunen, J., 1993 "Acousto-optic conversion of laser beams into flat-top beams", *J. Mod. Opt.* 40, 625-635.
- [9] Vazquez, R.A., Vachss, F.R., Neurgaonkar, R.R., Ewbank, M.D., 1992 "Large photorefractive coupling coefficient in a thin cerium-doped strontium barium niobate crystal", *J. Opt. Soc. Am. B* 8, 1932-1941.
- [10] Zebda, Y., Qasaimeh, O., 1993 "Frequency and time response of PIN photodiode", *J. Opt. (India)* 22, 85-91.
- [11] Rashed, A.N.Z., 2014 "Best candidate materials for fast speed response and high transmission performance efficiency of acousto-optic modulators", *Opt. Quant. Electron.* 46, 731-750.
- [12] Vachss, F., Michael, I. Mc., 1992 "Observation of high-gain nonlinear acousto-optic amplification", *Opt. Lett.* 17, 453-455.
- [13] Neogi, A., Maheswari, K.P., Shodha, M.S., 1994 "Modulation instability in optically strained magnetoactive semiconductors", *J. Opt. Soc. Am. B* 11, 597-604.
- [14] Amina, M., Ema, S.A., Mamun, A.A., 2017 "Higher order nonlinear dust-acoustic waves in a dusty plasmawith dust of opposite polarity", *Chin. J. Phys.* 55, 619-629.
- [15] Artemenko, O.L., Sevruck, B.B., 1995 "Parametric interaction of electromagnetic and acoustic waves in cubic semiconductors with strain dependent dielectric media", *Phys. Stat. Sol. (b)* 189, 257-264.
- [16] Sen, P., 2001 "Improvement in figure of merit by doping for squeezed state generation in GaAs", *J. Nonlinear Opt. Phys. Mater.* 10, 377-387.
- [17] Allevi, A., Bondani, M., Ferraro, A., Paris, M.G.A., 2006 "Classical and quantum aspects of multimode parametric interactions", *Laser Phys.* 16, 1451-1477.
- [18] Singh, M., Aghamkar, P., Kishore, N., Sen, P.K., 2008 "Nonlinear absorption and refractive index of Brillouin scattered mode in semiconductor-plasmas by an applied magnetic field", *Opt. Laser Tech.* 40, 215-222.
- [19] Parsons, F.G., Chen, E. Yi, Chang, R.K., 1971 "Dispersion and nonlinear optical susceptibilities in hexagonal II-VI semiconductors", *Phys. Rev. Lett.* 27, 1436-1439.
- [20] Fong, C.Y., Shen, Y.R., 1975 "Theoretical studies on the dispersion of the nonlinear optical susceptibilities in GaAs, InAs, and InSb", *Phys. Rev. B* 12, 2325-2328.
- [21] Alexandru, P., 2003 "Nonconventional calculation of the second order susceptibility in polar semiconductors", *J. Phys.: Condens. Matter* 15, L559-L564.
- [22] Anderson, D.R., 1988 "Surface wave excitation at the interface between diffusive Kerr-like nonlinear and linear media", *Phys. Rev. A* 37, 189-195.
- [23] Vartharajah, P., Moloney, J.V., Newell, A.C., Wright, E.M., 1993 *J. Opt. Soc. Am. B* 10, 46.
- [24] Lal, B., Aghamkar, P., Kumar, S., Kashyap, M.K., 2011 "Second order optical susceptibility in doped III V piezoelectric semiconductors in the presence of a magnetostatic field", *Eur. Phys. J. D* 61, 717-724.
- [25] Guha, S., Ghosh, S., 1977 "Convective instability in n-InSb in the presence of electric and magnetic fields", *Phys. Stat. Sol. (a)* 41, 249-254.
- [26] Yariv, A., 1997 "Quantum Electronics in Modern Communications", Oxford University Press, New York, p. 479.
- [27] Yariv, A., 1975 "Quantum Electronics", Wiley, New York, p. 158.