

Fractional Order Differentiator Based Filter For Edge Detection of Low Contrast Underwater Images

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Abstract—Underwater images usually suffer from degraded visibility. Light attenuates and scatters in water resulting in low contrast and haziness in the scenes. Therefore, the main problems to be dealt with in underwater environment are poor contrast, non-uniform lighting, haziness and blurring. Hence, in order to study underwater images, it becomes utmost important to extract the invisible or unclear edges. This paper presents an edge detection method by using fractional order differentiation (FOD) approach. As texture plays a major role in low-level image analysis, therefore texture based image enhancement is very important. In order to attain texture enhancement in images, an algorithm based on the Grünwald-Letnikov (G-L) fractional order derivative is proposed. Considering the G-L based fractional differential operator's basic definition and implementation, a filter is devised and its applicability for texture enhancement is analyzed. Legendre polynomials based FOD has been used to design the filter. Further High Pass Filters (HPF) and Low Pass Filters (LPF) with the concept of intensity factor (γ) are designed. Next, a multiplication operation is performed in the pre processing stage. At last, Sobel's method for edge detection is applied on the resultant pre processed image. The algorithm is experimented on various underwater images. The quality of the resultant images are evaluated by checking their respective Mean Square Error (MSE) and Peak Signal to Noise Ratio (PSNR) values. Further, the results are compared with another approach based on Riemann Liouville (R-L) fractional differential operator. The analysis of tests proves that the proposed method displays better results for detecting edges of low contrast underwater images and reveals more information than Histogram Equalization method and R-L method based on MSE and PSNR values.

Keywords— Fractional Order Differentiator, Legendre polynomials, Grünwald Letnikov fractional derivative, MSE, PSNR, Sobel Edge Detector, Underwater Images.

I. INTRODUCTION

Underwater imaging raises new difficulties and intrudes considerable challenges due to absorption

and scattering effects of light. Thus, exploration and investigation of underwater images have gained utmost importance in the past few years. Underwater images suffer from low contrast and degraded vision of farther objects due to the debilitation of the propagated light as the distance increases from the camera. When such images are captured, quality of the image derogates because of various factors like dissolved organic matters, small observable floating particles i.e. marine snow, ripples formed in water, unavailability of lighting etc. [1]. Therefore the image is captured via camera from a small distance in order to maintain a suitable quality. Further environmental conditions like light changing, turbidity in water, and color diminishing as we go deeper in water also adds to the haziness in images [2]. Hence it is necessary to preprocess the underwater images before applying high level image processing techniques like restoration and enhancement [3, 4].

Smoothness and texture edges are the major dominant factors present in underwater images. Due to the smoothness property of underwater images, the LPF operations are applied which extricates their texture edges. The edge, being the basic feature of the image, contains most of the information of images [5]. The so-called edge refers to the change in gray level of the surrounding pixels set, which can exist between the object and background or the target object and the other object [6]. The traditional edge detection algorithms however determine edges in some cases, but also have the edge information missing and may sometime showcase false edges. However, till now no universal algorithm has been developed which can detect all scales of edges. In 1971, the edge detection algorithm was introduced by Rosenfield for the very first time [7]. Martin et al. at Berkeley University implemented that if the results after applying edge detection are consistent

with human vision then it is a good edge detection [8, 9]. In this paper, a new methodology is implemented to extract image edges using fractional order approach such that they resemble more uniformly with human vision. The texture edges have been obtained by convolving the original image with the designed filters [10]. Generally, Canny's criterion is taken as an ideal standard to estimate the precision of edge detection algorithm. Canny's method of edge detection is based on derivatives of the Gaussian [11]. The other traditional edge detection algorithms are Roberts method, Sobel method, Prewitt operators, FIR filters, recursive filters, Deriche filter, first order derivative of Gaussian function etc. [11, 12, 13, 14].

Some of the popular edges preserving filtering techniques used for underwater preprocessing are Homomorphic filtering, Anisotropic diffusion, Relaxation labeling methods, differentiation based logarithmic image processing (LIP) models, Wavelet fractional differentiation and contrast-based techniques. Jie Hou et al [15], describes the application of wavelet transform with canny edge detector. [16, 17, 18, 19, 20, 21, 22].

By visual interpretation, it is noticed that texture features are sharp details in images and it is inferred that differential operators may be considered for highlighting the textural information in images. Basically, first and second order differential operators such as Gradient and Laplacian operators are used to highlight the edges and boundaries in images. It was also proved in the recent literature that fractional differential operators [23] are found to be more appropriate for image textural features than integral differential operators. Thus, fractional differential operators are considered for texture enhancement in images. The Fractional order differentiation based filter (FOD) design is modeled using Legendre polynomials [24]. The method can be used to calculate the FOD of noise-free as well as noisy signals.

The basic summary of this paper is detailed as reflected: The design of Legendre Polynomial based FOD filter is described in Part 2. Further, the designed algorithm based on the G-L derivative definition for edge detection is reviewed in Part 3. Details of experiment are showcased in Part 4. Experimental results using the proposed method on test images are obtained in Part 5. At last, conclusion is discussed in Part 6.

II. DESIGN OF FILTER

A. Legendre Polynomials

Legendre polynomials (or Legendre functions) are a particular class of functions which are distinctly convenient for approximating other functions. It plays a key role in many problems of numerical integration, numerical differentiation, numerical solution of ordinary polynomials, uniform approximation, partial approximation and least square approximation of continuous functions and implied calculation of such approximation.

In mathematics, Legendre functions are solved from the following Legendre's differential equation:

$$\frac{d}{dx} \left[(1-x^2) \frac{d}{dx} P_n(x) \right] + n(n+1) P_n(x) = 0 \quad (2.1)$$

Or

$$(1-x^2)P'' - 2xP' + n(n+1)P = 0 \quad (2.2)$$

where $n \in R$ is called a *Legendre equation* of order n .

The Legendre polynomials $P_n(x)$ for $n=1,2,3...$ can be determined using Rodrigue's Formula as:

$$P_n(x) = \frac{1}{2^n n!} \frac{d^n}{dx^n} (x^2-1)^n \quad (2.3)$$

Therefore, for individual values of n , Legendre polynomials of distinct degrees are obtained using Rodrigue's Formula.

$$P_0(x) = 1$$

$$P_1(x) = x$$

$$P_2(x) = \frac{1}{2}(3x^2-1)$$

and so on.

The graphs of these polynomials (upto $n = 5$) are shown in Fig.1.

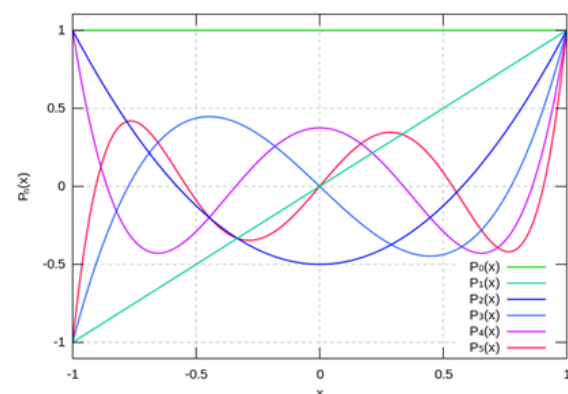


Fig. 1. Graph of Legendre's Polynomial



These polynomials of distinct orders can be used to approximate / fit any given function. This property will be used to define the proposed filter coefficients as shown in next part.

B. Legendre Polynomials based algorithm

Consider two higher order functions as $X(t)$ and $\tilde{X}(t)$, which are differentiable in real range.

$X(t)$ is measured function. $\tilde{X}(t)$ is given function.

Thus, the measured function can be written as:

$$X(t) = \tilde{X}(t) + e(t) \quad (2.4)$$

Where $e(t)$ is the approximate error.

The present work comprehends smoothing of measured function by estimating the dn^{th} derivative, t point window filter and a n -degree polynomial. Using the Savitzky-Golay filter, any least square polynomial can be obtained by the following expansion [25, 26]:

$$X(t) = \sum_{k=0}^n z_k Q_k(t) \quad (2.5)$$

Where $X(t)$ is a polynomial function of degree- n , which is used to approximate the given signal, $t = 1, 2, 3, \dots, L$ is the region of t^{th} point in the filtering window and z_k is the k^{th} coefficient of the polynomial. The coefficients z_k can be obtained by using least square method. Q_k denotes equivalent value of Legendre polynomial respective to t^{th} point and k^{th} coefficient.

Equation (2.5) can be expanded as:

$$\begin{aligned} Q_0(1)z_0 + Q_1(1)z_1 + Q_2(1)z_2 + \dots + Q_n(1)z_n &= x_1 \\ Q_0(2)z_0 + Q_1(2)z_1 + Q_2(2)z_2 + \dots + Q_n(2)z_n &= x_2 \\ Q_0(3)z_0 + Q_1(3)z_1 + Q_2(3)z_2 + \dots + Q_n(3)z_n &= x_3 \\ \dots &\dots \\ Q_0(L)z_0 + Q_1(L)z_1 + Q_2(L)z_2 + \dots + Q_n(L)z_n &= x_L \end{aligned} \quad (2.6)$$

For better understanding, we will use matrix notation further, and (2.6) can be re-written as

$$X = QZ + e \quad (2.7)$$

Where $X = [x_1, x_2, \dots, x_L]^T$ denotes the observed function points in the filtering window matrix, $Z = [z_1, z_2, \dots, z_L]^T$ denotes the coefficient matrix and

e is the estimated error. Q is a Legendre polynomial matrix of order $L \times (n+1)$ and can be defined as

$$Q = \begin{bmatrix} Q_0(1) & Q_1(1) & Q_2(1) & \dots & Q_n(1) \\ Q_0(2) & Q_1(2) & Q_2(2) & \dots & Q_n(2) \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ Q_0(L) & Q_1(L) & Q_2(L) & \dots & Q_n(L) \end{bmatrix} \quad (2.8)$$

The values of components of Q can be computed by using equation (2.3). Thus the components of the matrix Q are the values of Legendre polynomial at specific point. Vector Z , storing the coefficients of the best-approximated polynomial, can be obtained as:

$$Z = (Q^T Q)^{-1} Q^T X \quad (2.9)$$

Further, equation (2.7) can be solved using equations (2.3) and (2.9). The estimation of given function is computed as:

$$\tilde{X} = QZ = Q(Q^T Q)^{-1} Q^T X = WX \quad (2.10)$$

where W symbolize the window's coefficient matrix, which can be employed for smoothening of the given function. Different window coefficient matrix can be utilized for different smoothing.

The **Grunwald-Letnikov (G-L)** fractional order derivative [27] can be expressed as

$${}_a D_t^\alpha f(x) = \lim_{h \rightarrow 0} \frac{1}{h^\alpha} \sum_{j=0}^{\left[\frac{x-a}{h}\right]} (-1)^j \binom{\alpha}{j} f(x-jh) \quad (2.11)$$

Where $\binom{\alpha}{j}$ is the binomial coefficient and a is the initial value. By contrast, the G-L derivative starts with the derivative instead of the integral. Being an adept method to compute fractional order derivative, it gives good approximation to fractional derivative for sufficiently lower value of h . Also it can be verified that precision of this method is high.

The FOD, corresponding to window coefficient matrix W , can be obtained by equation (2.11). By applying various properties of FOD on equation (2.10), we obtain,

$$\tilde{X}_t^\alpha = Q_t^\alpha Z = W_t^\alpha X = z(Q^T Q)^{-1} Q^T X \quad (2.12)$$



Where \tilde{X}_t^α indicates the α^{th} derivative of the t^{th} point in the window filter and W_t^α expresses the α^{th} derivative coefficient vector of the t^{th} point in the window filter.

III. ALGORITHM FOR DETECTION OF EDGES

The proposed algorithm is divided mainly into two parts, Algorithm 1 and Algorithm 2. Algorithm 1 is defined for the filter designing by deriving window coefficient matrix and Algorithm 2 is defined for detecting edges.

A. Algorithm 1

Input arguments: L, n, α

Output arguments: W, h_0

L : Differentiator Filter Length

n : Polynomial Order

α : Derivative Order

Q : Legendre polynomial matrix

z : Constant value

W : Window Matrix

Γ : Gamma Function

m : integer value such that $\alpha = m + \gamma$, and $0 < \gamma \leq 2$.

start

for $a \leftarrow 1$ to L

for $b \leftarrow 0$ to n

Enumerate matrix Q_{ab}

$$Q_{ab}(a) = 2aQ_b(a) - Q_{b-1}(a)$$

{ Here $Q_0(a) = 1$, $Q_1(b) = b$ }

end

end

for $a \leftarrow 1$ to L

for $b \leftarrow 0$ to n

$$\text{Compute } z = \frac{\Gamma(n+1)\Gamma(m+1)}{\Gamma(n+1-\alpha)} a^{n-\alpha}$$

$$W_t^\alpha = z(Q^T Q)^{-1} Q^T$$

$$G_0(a) \leftarrow W_t^\alpha$$

$$h_0(a) = (-1)^L G_0(a)$$

end

end

end

B. Algorithm 2

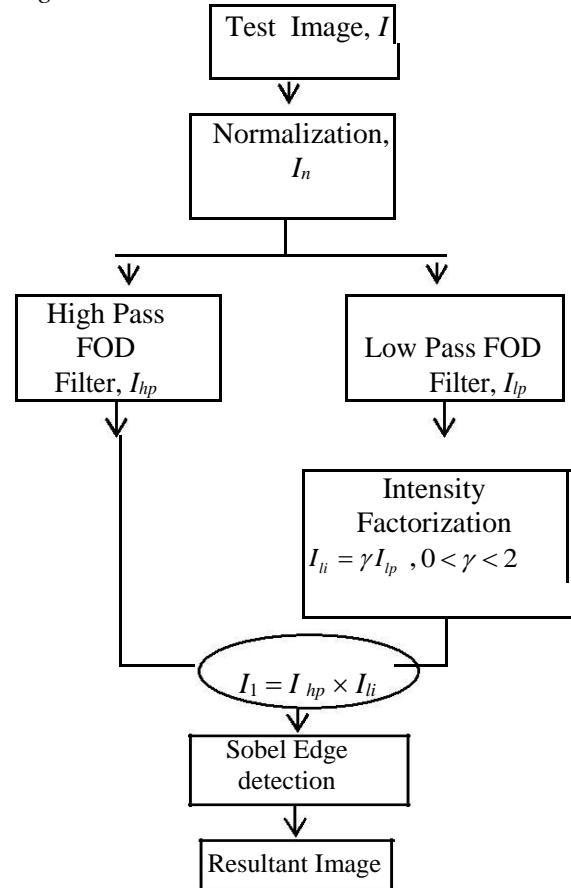


Fig. 2. Flow Chart of processed Algorithm 2

Fig. 2 demonstrates the flow chart of the proposed edge detection algorithm method. The test image, I , has to be normalized initially. The normalized image, I_n , is then processed with the designed FOD filter, created using Legendre polynomial. The LPF and HPF filtering operations are performed on the normalized image separately. I_{li} symbolizes the output image of the LPF filtering operation followed by the intensity factor. The intensity factor, γ , varies from 0 to 2. I_{hp} signifies the output image of the HPF filtering operation. Multiplication operation has been performed between I_{li} and I_{hp} . At last, for detecting the edges, the traditional sobel edge detection algorithm is applied on this pre processed image.

I : Test Image

γ : Intensity Factor

I_n : Normalized image

I_{lp} : Extracted image with LPF

I_{hp} : Extracted image with HPF

I_1 : Unprocessed Image

I_E : Resultant Pre-processed Image

$h_0 ()$: High pass filtering operation

$G_0 ()$: Low pass filtering operation

$Sobel ()$: Standard sobel edge detection algorithm

$Norm ()$: Normalization of image

start

$I_n = Norm (I)$

$I_{hp} = h_0 (I_n)$

$I_{lp} = G_0 (I_n)$

$I_{li} = \gamma I_{lp}$

$I_1 = I_{hp} * I_{li}$

$I_E = Sobel (I_1)$

end

IV. EXPERIMENTS

The suggested algorithm is implemented on two test images. Both the test cases being taken are underwater images having smoothness properties as well as low contrast.

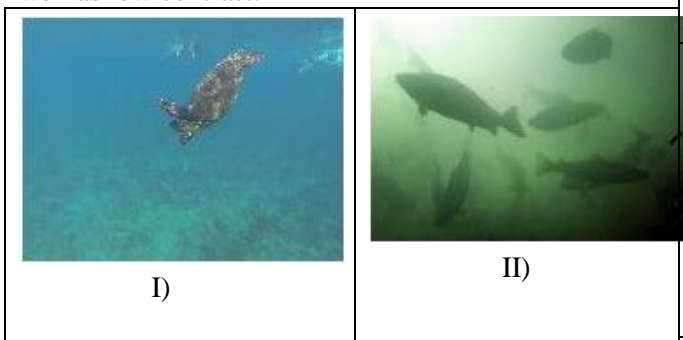


Fig. 3. Experimental Images: I) Test Image A (1200x900 pixels), II) Test Image B (600x420 pixels)

V. RESULTS

a) Result of proposed algorithm

The newly built algorithm is experimented on the test images shown in Fig. 3. The results acquired for Test images A and B have been shown in Fig.4 and 5 respectively. The values of differentiator order and

length of the filter being taken are $\alpha = 0.3$ and $L = 9$ respectively. Table 1 represents the use of fractional order filter with Sobel algorithm. Further using Sobel Operator with the fractional order, it is analyzed that the lowest values of Mean Square error are obtained at order 0.3. This means it gives the highest range of Peak signal to noise ratio values at the same order, since MSE and PSNR are inversely proportional to each other. Thus, when the order is near 0.3, we will obtain a minimum error between the original and observed image, i.e. the obtained observed image will be more close to the original image. Also, when the differential order is smaller, the enhancement is more and in case of edge detection, the edges being obtained are more crisp and clear.

Further, the algorithm is validated for four different values of intensity factor (γ). The value of factor coefficients is $\gamma = 0.3, 0.6, 0.9$ and 1.5 . It is being observed that more texture edges are enhanced for lower value of intensity factor.

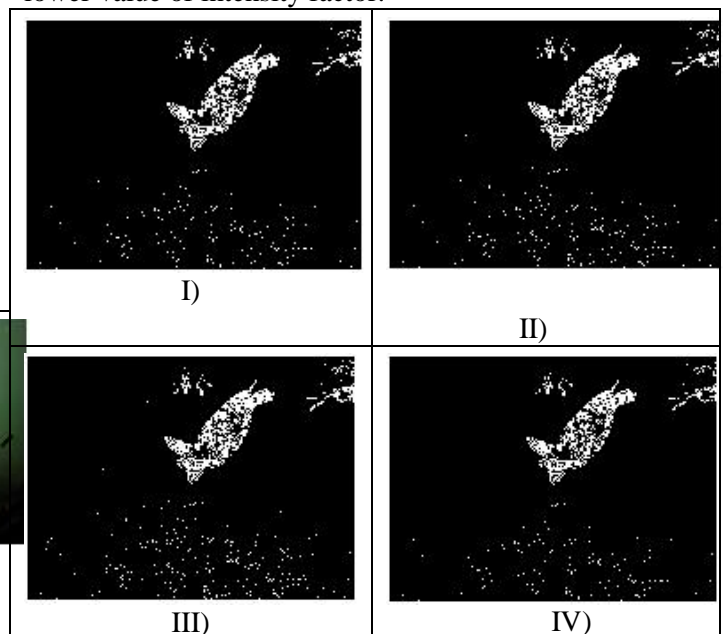


Fig. 4. Detected edges of Test Image A by proposed algorithm at I) $\gamma = 0.3$, II) $\gamma = 0.6$, III) $\gamma = 0.9$, IV) $\gamma = 1.5$



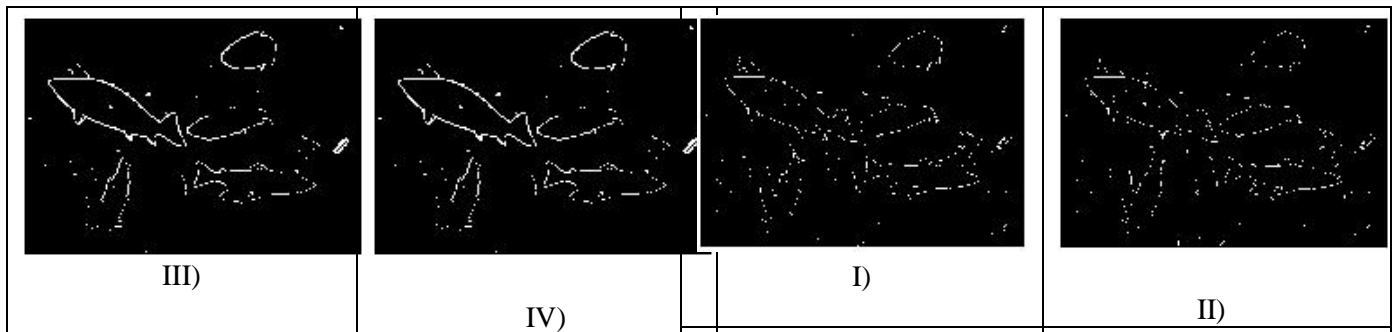


Fig. 5. Detected edges of Test Image B by proposed algorithm at I) $\gamma=0.3$, II) $\gamma=0.6$, III) $\gamma=0.9$, IV) $\gamma=1.5$

b) Comparison with other methods

Fig. 6 and Fig. 7 exhibits the result attained for Test image A and B respectively by prevailing edge detection methods.

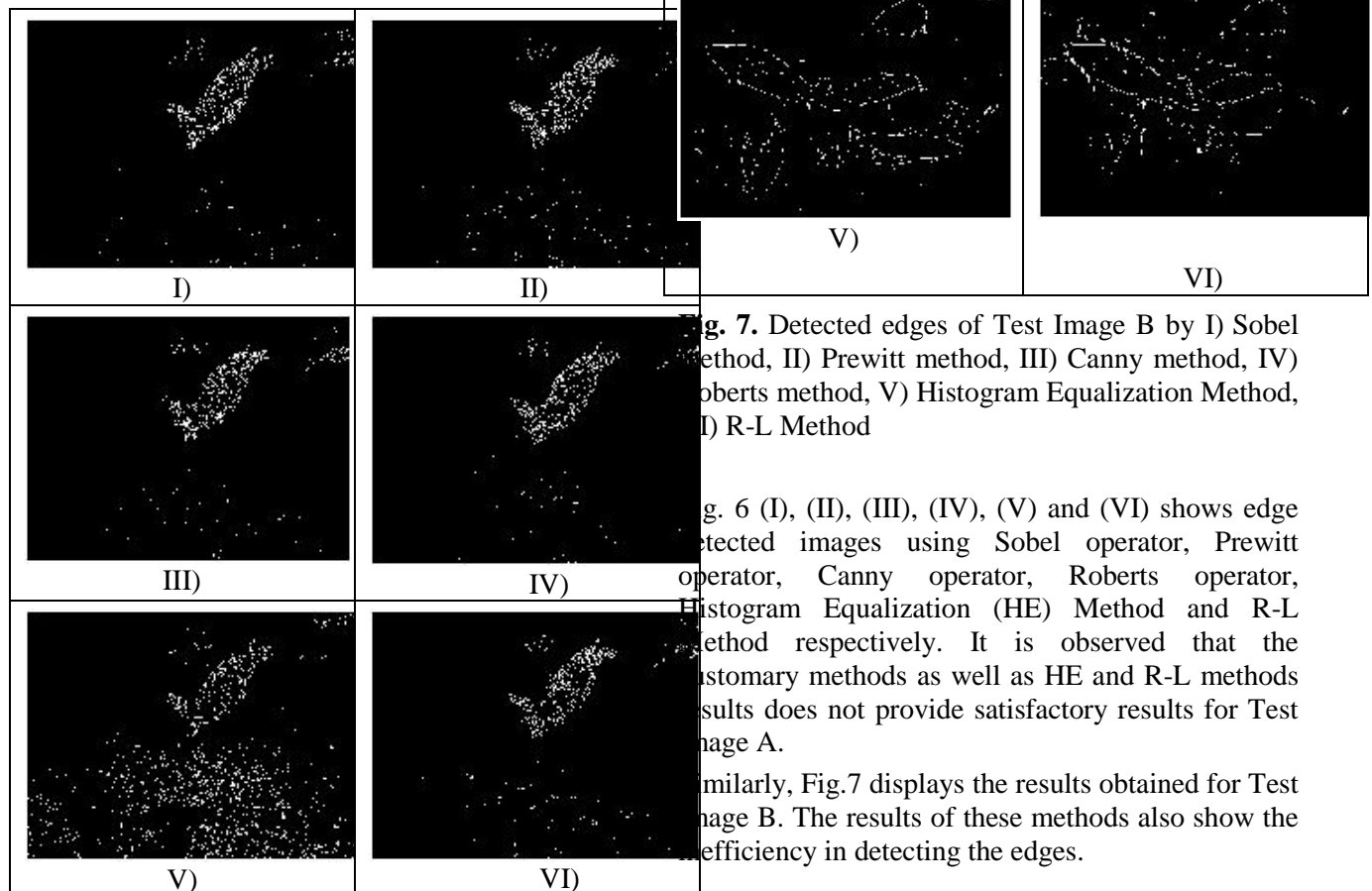


Fig. 7. Detected edges of Test Image B by I) Sobel method, II) Prewitt method, III) Canny method, IV) Roberts method, V) Histogram Equalization Method, VI) R-L Method

Fig. 6 (I), (II), (III), (IV), (V) and (VI) shows edge detected images using Sobel operator, Prewitt operator, Canny operator, Roberts operator, Histogram Equalization (HE) Method and R-L method respectively. It is observed that the customary methods as well as HE and R-L methods results does not provide satisfactory results for Test image A.

Similarly, Fig.7 displays the results obtained for Test image B. The results of these methods also show the efficiency in detecting the edges.

Fig. 6. Detected edges of Test Image A by I) Sobel method, II) Prewitt method, III) Canny method, IV) Roberts method, V) Histogram Equalization Method, VI) R-L Method

VI. CONCLUSION

Based on the proposed methodology, it is found out that edge extraction in low contrast underwater images having smoothness property is strenuous. The inbuilt standard algorithms for edge detection



fail to provide satisfactory results in these types of images. The newly designed filter, based on G-L definition, plays a major role in the designed algorithm. The pre processed image using FOD based approach followed by Sobel edge detector performed better than all the traditional algorithms. Further, on the basis of image quality assessment using MSE and PSNR values, it has been observed that the proposed algorithm also displayed better results than the Histogram Equalization (HE) or Riemann-Liouville (R-L) methods.

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Table 1: Sobel operation using fractional order operator

α	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
Image A									
MSE	74.7765	73.8648	72.2873	73.9965	74.8763	75.6090	76.1843	78.0956	81.0121
PSNR	29.0314	29.2432	29.2650	29.1911	29.0678	28.6744	28.0775	26.9976	21.7654
Image B									
MSE	106.5434	105.6567	104.0995	105.8103	106.7690	107.4998	107.9965	109.7121	113.9870
PSNR	23.5383	23.6299	23.6654	23.5984	23.4002	22.7640	22.0921	19.2251	14.6209