Academic Science

International Journal of Electronics, Electrical and Computational System
IJEECS
ISSN 2348-117X
Volume 6, Issue 7
July 2017

Memristor Models: A Review

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Abstract – This paper is a review paper which shed light on the introduction and modelling of memristor. This study mainly concentrates on memristor modelling. The memristor models are reviewed in terms of their effectiveness and accuracy to describe the behaviour of memristor. The current voltage relationship and state variable in these models are analyzed and discussed.

Keywords

Memristance, linear ion drift model, non linear ion drift model, Simmons tunnel barrier model and TEAM model.

1. Introduction

Every electronics engineer is acquainted with three basic electronic circuit elements which are named as the resistor, the capacitor, and the inductor. These basic elements can be described in the terms of two basic circuit variables – current, voltage, charge and flux .The great researcher and circuit theorist, Leon Chua first proclaimed in 1971 that there exists a fourth fundamental element which can be characterized by the relation between flux and charge. He called the fourth basic circuit element the memristor. He first introduced the mathematical concept and realization of memristor in 1971 in his seminal paper [1].Chua gave a mathematical relation between flux and charge which represents the device named memristor. The proposed relation between charge and flux was given by $g(\phi, q) = 0$ where ϕ is flux and q is charge. Moreover, he presented an electromagnetic interpretation of the memristor characteristics [1] .Two types of memristors were defined by Chua in his paper. They are called charge-controlled and flux-controlled memristor. The voltage across a charge-controlled memristor is given by:

$$v(t) = M(q(t))\, i(t) \qquad (1)$$

$$M(q) = d\phi(q)/dq \qquad (2)$$

Similarly, the current of a flux-controlled memristor is given by:

$$i(t) = W(\phi(t))\, v(t) \qquad (3)$$

$$W(\phi) = dq(\phi)/d\phi \qquad (4)$$

where $M(q)$ and $W(q)$ are the memristance and memconductance respectively. In 1976, a paper was published by Chua and his student, Kang to describe memristive systems [2]. A generic equation was formulated in this paper to describe the behavior of memristor. This equation is

$$y = g(x,u,t)\, u \qquad (5)$$

$$dx/dt = f(x,u,t) \qquad (6)$$

where x is the state variable, u and y are the input and the output of the system, respectively, f is a continuous n-dimensional function and g is a continuous scalar

function.

Memristor stores the value of resistance (memristance) when power is switched off and offers the same value of resistance to current when voltage is applied again. Thus, it stores the resistance. The total memristance of memristor can be increased or decreased by changing the polarity of applied electric field. Pinched hysteresis or double loop curve is the fingerprint of memristor. Memristors can be categorised according to the types of charge carriers and materials used. Each type of memristor has its own material properties, switching mechanisms and applications. Scientists from all over the world proposed different models of memristor. They are linear ion drift model , Non-linear ion drift model , Simmon tunnel barrier model and TEAM model . There are various digital (for instance, Non-volatile memories, logic gates) and analog (eg: filter design, oscillators, amplifiers) applications of memristor. Memristor also plays a very vital role in the field of neuromorphic engineering.

This paper has been categorised in three sections, include the current introductory section.1. Thereafter, section.2 incorporates the information of

different memristor models along with their advantages and limitations in brief. Finally, section.3 draws necessary conclusions.

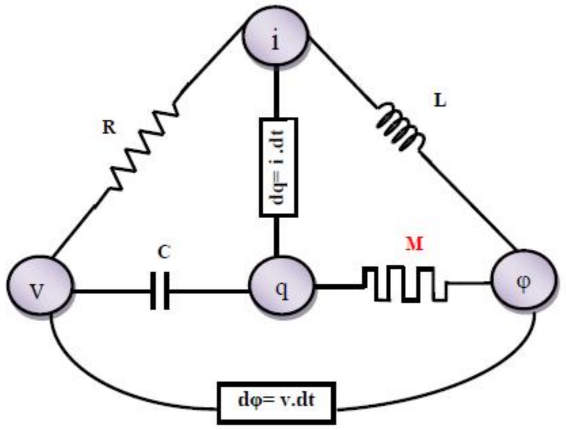

Figure 1: Linkage between four fundamental circuit elements[10].

2. Memristor Models

As the properties of memristor can be utilized to realize many circuits, so it is necessary to give proper models for the design and analysis of memristor based circuits. In 2008, HP labs first put forward practical model of memristor, later then scientists from all over the world proposed many models of memristor. In this section the four models of memristor are reviewed in brief.

3.1. Linear Ion Drift Model

Linear ion drift model [3] was presented by HP team for analyzing and describing the behaviour of titanium dioxide based memristor which is also called HP memristor. This model takes into account the fact that two series variable resistors RON and ROFF represent the resistances of doped and undoped region respectively. Let D is the length of physical device and w is the width of doped region, then memristance of the device at any instant t is given by:

$$M(t) = RON\frac{w(t)}{D} + ROFF\left(1 - \frac{w(t)}{D}\right) \qquad (7)$$

$$\frac{dw}{dt} = \frac{\mu\, RON}{D} i(t) \qquad (8)$$

where μ is the average ion mobility and w is the state variable as it changes depending on the charge passing through the memristor at time t.

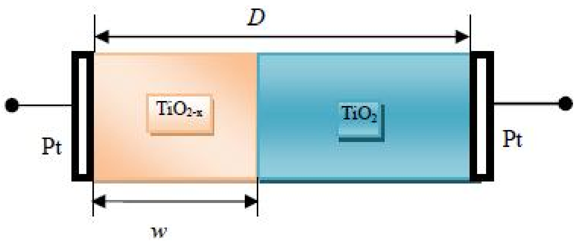

Fig. 2.2 Schematic of HP memristor. [10]

This model has the advantages of being simple. But it also suffers from disadvantage of being reduced non-linearity current-voltage curve. The model is based on the assumption that vacancies can freely move along the entire length of the device and electric field applied is uniform.

2.2. Non Linear Ion Drift Model

The hysteresis behaviour of memristor can be described very well with the help of linear ion drift model [4]. But this model suffers from certain limitations which are related to basic electrodynamics. It has been proved by the studies and experiments that the characteristics and behaviour exhibited by the memristor are non–linear to a great extent. Linear ion drift model has very low accuracy and it has certain drawbacks. Non linear ion drift model has some postulates which are: memristor is a voltage controlled element in which there is a non-linear relationship between the voltage and the state derivative and switching behaviour is asymmetric. Lehtonen et al proposed a model based on the results of [5] . The mathematical equations for describing the current–voltage relationship of this model can be given by:

$$i(t) = w^n(t)\beta \sinh(\alpha v(t)) + \chi[\exp(\gamma v(t)) - 1] \qquad (9)$$

The equation for state variable derivative is:

$$dw(t) / dt = avm(t) f(w) \qquad (10)$$

where α, β, γ, χ, a and m are experimental fitting parameters, and n predicts the ability of state variable to affect the current. Here, the state variable w is normalized within the interval [0, 1]. This model has exponential relation between current and voltage which presents a logical description of memristor. This model is more sensitive to voltage levels due to which it is flexible to reconcile stable reading with fast writing.

2.3. Simmons Tunnel Barrier Model

Simmons tunnel barrier model [6] has more accuracy to describe the behaviour of memristor. This model considers that resistor is connected in series with electron tunnel barrier for modeling

memristor. In linear ion drift model and non linear ion drift model, memristor has been modeled as the two resistors presenting doped and undoped region which have end to end connections. Nonlinear and asymmetric switching behavior are considered to analyse simmons tunnel barrier model because the kinematics of the ionized dopants can be better judged by exponential relation. Simmons tunnel barrier's width x is chosen as the state variable [7]. The equation showing the dependence of current on voltage is ambiguous [6] and can be given as:

$$i(t)=\tilde{A}(x,v_g)\varphi_1(v_g,x).\ exp(-B(v_g,x).(\ \varphi_1^{0.5}(v_g,x))- \tilde{A}(x,v_g)(\varphi_1(v_g,x)+ev_g)\ .\ exp(B(v_g,x).\ (\varphi_1(v_g,x)+ev_g)^{0.5} \quad (11)$$

$$v(g)=v-i(t)R_s \quad (12)$$

The equation for state variable derivative is as follows:

$$\frac{dx}{dt}=f(x)=\begin{cases} coff\sinh\left(\frac{i}{ioff}\right)\exp\left[-\exp\left(\frac{x-aoff}{wc}-\frac{i}{b}\right)-\frac{x}{wc}\right] & i>0 \\ con\sinh\left(\frac{i}{ion}\right)\exp\left[-\exp\left(\frac{x-aon}{wc}-\frac{i}{b}\right)-\frac{x}{wc}\right] & i<0 \end{cases} \quad (13)$$

where Coff, Con, aoff, aon, ioff, ion, and b are fitting parameters. The order of con is 2.4. . ion and ioff do not allow the current threshold to move in an ineffective way. These current thresholds play a vital role in applications of digital electronics .There is no need of window function in this model as coff and con confine the lower and upper bound for x, respectively. This model has certain limitations; (1) it is complex in terms of analysis, (2) the relationship between current and voltage is implicit, (3) it has low computational efficiency and (4) it is not a generic model, since it is suitable for particular type of memristor. In [8], SPICE model of Simmons tunnel barrier model has been presented.

2.4. Threshold Adaptive Memristor Model

A more general, simple and flexible model was proposed by Kvatinsky [9] which is called as threshold adaptive memristor model. This model is close to Simmons tunnel barrier model in terms of accuracy. The mathematical functions and expressions employed in TEAM model are simpler and less complex than Simmons tunnel barrier model. However, TEAM model represent the same physical behaviour as described by Simmons tunnel barrier model. TEAM model is based on the assumptions that there is no change in the state variable below a certain threshold current, and state variable has a polynomial dependence. In this model, approximation of threshold current is used. The internal state derivative of this model depends on the current and state variable itself as mentioned in the below equations:

$$\frac{dx(t)}{dt}=\begin{cases} k_{off}\cdot\left(\frac{i(t)}{i_{off}}-1\right)^{\alpha_{off}}\cdot f_{off}(x), & 0<i_{off}<i \\ 0, & i_{on}<i<i_{off} \\ k_{on}\cdot\left(\frac{i(t)}{i_{on}}-1\right)^{\alpha_{on}}\cdot f_{on}(x), & i<i_{on}<0, \end{cases} \quad (14)$$

where k_{off}, k_{on}, α_{off}, and α_{on} are constants ($k_{off} > 0$, $k_{on} < 0$) and x is the state variable. i_{off} and i_{on} are current thresholds. The functions $f_{off}(x)$ and $f_{on}(x)$ act as the window functions, forcing x to the bounds [xon, xoff]. The relationship between the current and voltage can be governed by the following equations:

$$v(t)=\left[RON+\frac{ROFF-RON}{xoff\ -xon}(x-xon)\right].i(t)\ or \quad (15)$$

$$v(t)=RON.\exp\left(\frac{\varphi}{xoff-xon}(x-xon)\right).i(t) \quad (16)$$

where φ is a fitting parameter . RON and ROFF are memristances at bounds xon and xoff, respectively. This model is flexible and general. The model can fit the different memristive models. The derivative of the internal state variable can be fitted to any memristive device type. This model consumes low computational time. It also characterizes a variety of different models.

3. Conclusion

This paper reviews memristor models. Memristor is a basic electronic element which can be defined by a relationship between flux and charge. After a deep study of memristor, researchers have built different analog and digital devices which will soon give a strong competition to the devices made of transistors. Different models of memristor are described and analyzed in this paper. Linear ion drift model is the first model introduced for imitiating the behaviour of memristor. It has explicit current-voltage relationship. But it suffers from some drawbacks such as it has not only the limitation of reduced current-voltage curve , but also this model has lowest accuracy. Also, this model does not

satisfy the results of many experiments which show that current and voltage depend non-linearly on each other. Non ion linear drift model has a more complex state variable and its accuracy is also low. Simmons tunnel barrier model has the highest accuracy but the current voltage relationship is ambiguous. It requires much computational time because of the involvement of complex exponential mathematical expressions. It practically best explains the behaviour of memristor. But it is specifically suited for a particular memristive device. TEAM model is the answer of all the limitations imposed by the previous discussed models. TEAM model is a general model and has sufficient accuracy. The current – voltage relationship is also explicit in the threshold adaptive model.

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