

# Effect of Particles Gradient on Radial Displacement in a Rotating Disc

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**ABSTRACT:** The effects of varying  $\text{SiC}_p$  particle gradient (PG) on radial displacements in a Functionally Graded (FG) rotating disc as been investigated. The disc is assumed to be made of Al- $\text{SiC}_p$  composite. The  $\text{SiC}_p$  content in the aluminum matrix is varying according to power law. The distributions of radial displacements are computed for the FGM disc by increasing  $\text{SiC}_p$  particle gradient. It is concluded that that radial displacements in a FGM disc decreases with increase in  $\text{SiC}_p$  particle gradient (PG).

**KEY WORDS:** Particle gradient, FGM, rotating disc

## 1. INTRODUCTION

Rotating disc is an important component in industry [1]. Functionally Graded Material (FGM) is a material, in which content of the reinforcement varies along the directions [2]. Jahed et al [3] developed a design to reduce mass of a disc operating at a high temperature. Callioglu [4] investigated analytical solutions for rotating FGM disc subjected to external pressures. The material properties of the disc were assumed to vary according to power law. It is concluded that with the rise in gradient parameter, stresses in the disc change significantly.

## 2. DISTRIBUTION OF REINFORCEMENT AND MATERIAL PROPERTIES

Let us suppose a FGM (Al- $\text{SiC}_p$ ) disc ( $a = 0.04 \text{ m}$  and  $b = 0.1 \text{ m}$ ) rotating at a constant speed of 15000 r.p.m. The  $\text{SiC}_p$  content  $V(r)$  in the disc is varies along the radius as given by,

$$V(r) = V_b \left( \frac{r}{b} \right)^n \quad (1)$$

Where  $V_b$  is the value of  $\text{SiC}_p$  content at the outer radius of the disc and  $n$  is  $\text{SiC}_p$  gradation index.

The density ( $\dots$ ) and Young's modulus ( $E$ ) varies along the radius according to power law,

$$\dots(r) = \dots_b \left( \frac{r}{b} \right)^{n_1} \text{ and } E(r) = E_b \left( \frac{r}{b} \right)^{n_2} \quad (2)$$

The exponents  $n_1$  and  $n_2$  are gradation indices for density and young's modulus.

## 3. MATHEMATICAL FORMULATION

The compatibility equation between the strain rates ( $v_r$  and  $v_\theta$ ) is given by,

$$v_r = v_\theta + r \frac{dv_\theta}{dr} \quad (3)$$

For elastic deformations, stresses ( $\tau_r$  and  $\tau_\theta$ ) and strains are related as [5],

$$v_r = \frac{1}{E(r)} (\dot{r}_r - \epsilon \dot{r}_r) \quad (4)$$

$$v_r = \frac{1}{E(r)} (\dot{r}_r - \epsilon \dot{r}_r) \quad (5)$$

The equilibrium equation for a constant thickness rotating is as given below [5],

$$\frac{d}{dr} [r \dot{r}_r] - \dot{r}_r + \dots (r) \ddot{S}^2 r^2 = 0 \quad (6)$$

The constitutive Eqs. (3-5) along with equilibrium eq. (6) have been solved to find the radial displacements in the Disc.

The radial displacement ( $u_r$ ) in FGM disc can be calculated as given by,

$$u_r = \frac{r}{E(r)} (\dot{r}_r - \epsilon \dot{r}_r) \quad (7)$$

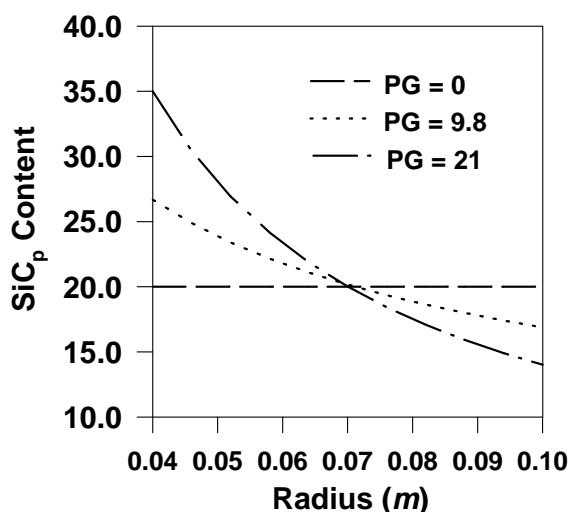
#### 4. RESULTS AND DISCUSSION

A code is developed using the analysis carried out in the last section. The effect of increasing particle gradient (Table 1) on distribution of radial displacements has been investigated.

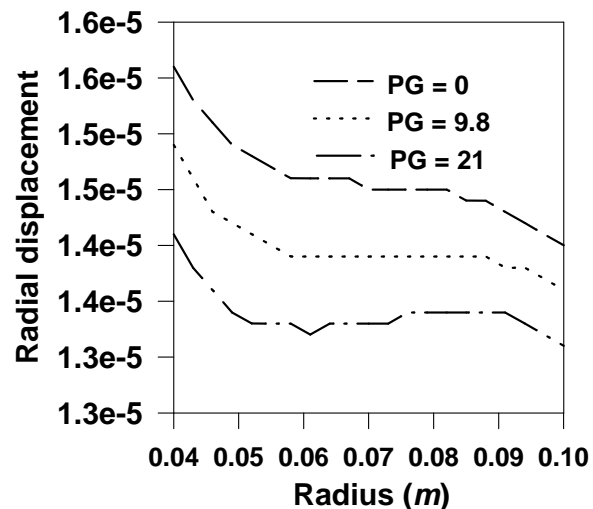
**Table 1:** Description of FGM discs

Disc	$n$	SiC <sub>p</sub> Content (vol %)			PG $V_i - V_o$
		$V_a$	$V_{av}$	$V_b$	
D1	0	20	20	20	0
D2	-0.5	26.66	20	16.86	9.8
D3	-1	35	20	14	21

Figure 1 shows the variation of SiC<sub>p</sub> Content in the FGM disc with increasing particle gradient. The average SiC<sub>p</sub> Content is same for all the three discs. It is observed that SiC<sub>p</sub> Content at the inner radius of the disc increases with the increase in particle gradient (PG). Similarly the SiC<sub>p</sub> Content at the outer radius decreases with the increase in particle gradient (PG). It is concluded that with the increase in SiC<sub>p</sub> particle gradient (PG) the radial displacement in the FGM disc decreases throughout.



**Fig. 1:** Variation of SiC<sub>p</sub> content in FGM discs



**Fig. 2:** Effect of particle gradient (PG) on Radial displacement in FGM discs.

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## 5. CONCLUSIONS

It is observed that by radial displacements are highest for the uniform composite disc and lowest for the FGM disc with maximum particle gradient (PG). It is concluded from the present study that the radial displacement decreases throughout for the FGM disc with the increase in particle gradient (PG).

## REFERENCES

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