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Effect of Uncertainty in Structural Parameters on Performance of Active Vibration Controller

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In this paper the effect of variations in mass and stiffness of a mechanical system on the performance of an active vibration controller is demonstrated. The mass and stiffness matrices are got from the free body diagrams of a four dof spring mass system. This system is then converted into a state space form. A negative velocity feedback strategy is used to control the vibrations of the system. Kalman filter is employed to estimate the states of the system to control first modal displacement. Stiffness and mass matrices are perturbed to simulate real life conditions. The effect of variation in mass and stiffness of system on control performance is studied and analysed. This system is then controlled with the controller based on original unperturbed model. Results show that performance of an active vibration controller gets appreciably deteriorated due to uncertainty in mathematical model of the system.

Keywords:* *Active vibration control, uncertainty in mathematical model, Kalman filter

1. Introduction

Space structures, aircrafts and vehicles are becoming lighter and faster. Demand of high performance vehicles, machinery and structures requires active control of unwanted vibrations in these flexible structures or machines. In most of these systems vibrations that need to be attenuated are in the low frequency range. Active Vibration Control (AVC) is suitable to control low frequency range vibrations encountered in such structures or machines. A typical AVC strategy uses mathematical model of the system and feedback control law. Finite Element (FE) method is a numerical method popularly used by engineers and researchers to get the discrete mathematical model of a system to predict the dynamic response for control purposes. Like all numerical methods it is an approximate method. In FE method the physical system is discretized into number of elements connected at points called nodes. Governing equations for each element are formulated and the values of field variable found at the nodal points. The values of the field variable are approximated inside the element assuming a polynomial variation in terms of the nodal values. The equations for all the elements are combined to get the equations of motion of the complete system. Then the appropriate boundary conditions are applied to get the equations of motion of the given system [1-3]. For undamped free vibrations the discretized mathematical model is written in the form:

$$\mathbf{M}\ddot{\mathbf{x}} + \mathbf{K} \quad = \mathbf{0} \qquad (1)$$

where $\mathbf{M}$ is the mass matrix, $\mathbf{K}$ is the stiffness matrix and $\mathbf{x}$ is the generalized displacement of the system under study. Assuming a solution of the form of $\boldsymbol{x} = \boldsymbol{\phi}\mathbf{s}\quad(\boldsymbol{\omega}\quad)$ and substituting in eq(1) results in the eigen value problem. Eigenvalue analysis is then carried out to obtain the dynamic response of the system in terms of its natural frequencies and mode shapes or eigen vectors.

$$[\mathbf{K} - \boldsymbol{\lambda} \quad]\boldsymbol{\phi} = \mathbf{0} \qquad (2)$$

where $\boldsymbol{\lambda} = \boldsymbol{\omega}^2$ is the eigen value, $\boldsymbol{\omega}$ is the natural frequency of vibration and $\boldsymbol{\phi}$ is the eigen vector. This model is then used in the control algorithm for AVC. Errors in calculating natural frequencies and mode shapes have a considerable effect in the control performance. Thus it can be seen that correct representation of mass and stiffness properties is critical for an active vibration controller. Errors in the mathematical model arise due to various reasons like discretization errors as a continuum is discretized into a finite element model, simplifying assumptions made during model formulation or errors arising due to the methods applied for numerical solution of the model. Due to these inherent errors in the mathematical modelling process the mathematical model representing the system differs from the actual system [4-7]. Uncertainty in parameters like thickness, modulus of elasticity, manufacturing variability or boundary conditions causes variation in the mass and stiffness

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matrices. Even mass produced and fabricated structures do not have the same dynamic response when same methods of manufacturing and assembly are employed. When AVC schemes are used on such structures or systems results vary from the desired ones. In this study the effect of variation in mass and stiffness on the performance of an AVC scheme is investigated. A four dof system is considered and three cases are studied. In first case it is assumed that the mathematical model is the correct representation of the actual system and it is controlled actively using a controller based on the same mathematical model. In the second case the stiffness matrix is perturbed to include effect of variation in dimensions, change in stiffness at the boundaries etc. This perturbed system is then controlled using a controller based on the original mathematical model. In the third case the mass matrix is perturbed and then this perturbed model is controlled using a controller based on the original mathematical model. Results show that errors in the mass and stiffness in the mathematical model affect the control performance significantly.

2. Mathematical formulation

Consider a four degree of freedom spring mass shown in fig.1. Equations of equilibrium for each mass are written from the free body diagrams and applying d'Alemberts principle to each mass (fig.2).

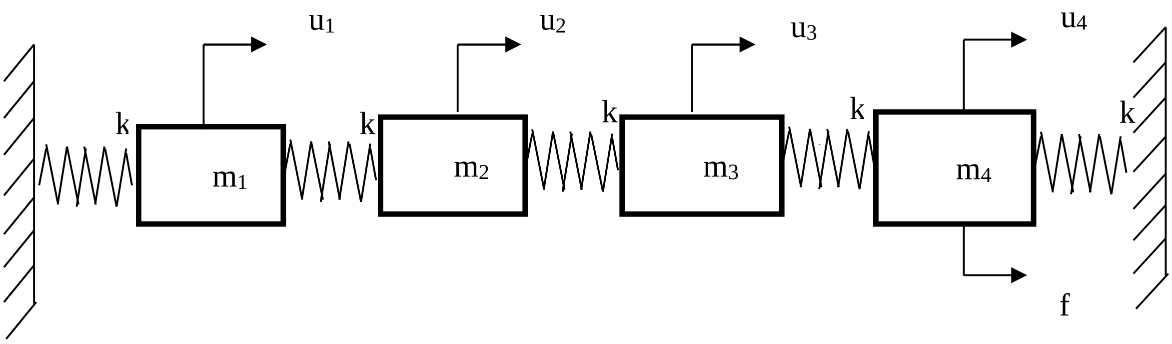

Figure 1: Four dof spring mass system

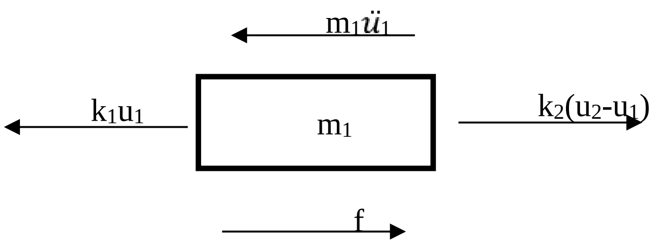

Figure 2: Free body diagram of mass m_1

$$m_1\ddot{u}_1 + (k_2 + k_1)u_1 - k_2 u_2 = 0 \quad \text{(3-a)}$$

$$m_2\ddot{u}_2 - k_2 u_1 + (k_2 + k_3)u_2 - k_3 u_3 = 0 \quad \text{(3-b)}$$

$$m_3\ddot{u}_3 - k_3 u_2 + (k_3 + k_4)u_3 - k_4 u_4 = 0 \quad \text{(3-c)}$$

$$m_4\ddot{u}_4 + (k_4 + k_5)u_4 - k_4 u_3 - f = 0 \quad \text{(3-d)}$$

Equation of motion of the system using direct method of assembly is given by

$$\begin{bmatrix} m_1 & 0 & 0 & 0 \\ 0 & m_2 & 0 & 0 \\ 0 & 0 & m_3 & 0 \\ 0 & 0 & 0 & m_4 \end{bmatrix} \begin{Bmatrix} \ddot{u}_1 \\ \ddot{u}_2 \\ \ddot{u}_3 \\ \ddot{u}_4 \end{Bmatrix} + \begin{bmatrix} k_1 + k_2 & -k_2 & 0 & 0 \\ -k_2 & k_2 + k_3 & -k_3 & 0 \\ 0 & -k_3 & k_3 + k_4 & -k_4 \\ 0 & 0 & -k_4 & k_4 + k_5 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ f \end{Bmatrix} \quad (4)$$

Assuming Rayleigh's damping and forced vibrations, these equations can be written in matrix form as:

$$[M]_4 \ \{\ddot{u}\}_{4X1} + [C]_4 \ \{\dot{u}\}_{4X1} + [K]_4 \ \{u\}_{4X1} = \{F\}_{4X1} \quad (5)$$

where $[C]$ is the damping matrix and $\{F\}$ is the force vector.

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3. Active vibration control

A negative velocity feedback control is employed to actively control first modal displacement of the four dof system under consideration. To apply the control law the system is converted into state space form as [8-10].

$$\{\dot{s}\}=[A]\{s\} + [B]\{U\} \tag{6}$$

where [A] is system matrix, [B] is control matrix,$\{s\}$ is state vector containing the four modal displacements & velocities and $\{U\}$ is the control force. An initial displacement of one unit is given to mass m_1. For controlling the first mode of vibration, modal force proportional to the negative of the first modal velocity is applied on mass m_4 as:

$$f_1 \; \square \; -\dot{\eta}_1 \tag{7}$$

Kalman filter is employed to estimate the states of the system. This estimated first modal velocity is multiplied by a suitable gain for calculating the control force [11].Kalman estimation equations are given as:

$$\{s_e\}_8 = [dA]_8 \; \{s_e\}_8 + [dB]_8 \; U + \{L_k\}_{8X1}(Y - \{C\}_{1X8}\{s_e\}_8) \tag{8}$$

$$\{s_e\}_8 = \{s_e\}_8 + \{M_k\}_{8X1}(Y - \{C\}_{1X8}\{s_e\}_8) \tag{9}$$

where '[dA]' &'[dB]' are discretized forms of [A] & [B] matrices respectively, '$\{L_k\}$' & '$\{M_k\}$' are Kalman gains, $\{s_e\}$ is the estimated state vector and 'Y' is the sensor signal. Newmark Beta method is used to simulate the actual response of the system to get 'Y'. To study the effect of perturbation in stiffness and mass matrix on the controller performance three cases are investigated as described below:

3.1 Case I

In the first case it is assumed that mathematical model is the true representation of the actual system. The AVC of the four dof system is simulated in software Matlab. Newmark Beta method is used to simulate the actual response of the system using a mathematical model of the system. This same mathematical model is used in the Kalman filter to estimate the states of the system for calculating control force.

3.2 Case II

In the second case the stiffness matrix is perturbed by 10% overall and 20% at the supports to reflect the variation in real life systems from the ideal ones. The first modal velocity is estimated using Kalman filter based on the original model. This first modal velocity is then used to control the vibrations of the perturbed model. The perturbed model representing the actual system is used in Newmark Beta method to simulate the response of the system to the control force applied on it.

3.3 Case III

In the third case the mass matrix is perturbed by 15% in the first & fourth mass and by 10% in the second & third mass. Again the negative velocity feedback control law is employed to control the vibrations of this perturbed system with the Kalman filter based on the original model. Newmark Beta method simulates the response based on the mathematical model in which mass has been perturbed. Table 1 tabulates the perturbations in the stiffness matrix and mass matrix for different cases. Table 2 lists the natural frequencies of vibration for the three cases.

Table 1: Mass and stiffness values for the original model and assumed experimental models

Original (case I)		Perturbed stiffnesses (Case II) (N/m)	Perturbed masses (Case III) (kg)
Mass (kg)	Stiffness (N/m)		
m_1= 1	k_1 =1000	k_1 = 1200	m_1= 1.15
m_2=2	k_2=1000	k_2= 1100	m_2=2.2
m_3=2	k_3=1000	k_3=1100	m_3=2.2
m_4=1	k_4=2000	k_4=2200	m_4=1.15
	k_5=500	k_5=600	

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Table 2: Natural frequencies of the four dof system

Mode	Frequency (Hz) (case I)	Frequency (Hz) (case II)	Frequency (Hz) (case III)
1	2.134	2.311	2.025
2	4.807	5.106	4.546
3	7.747	8.247	7.259
4	9.4675	9.9708	8.901

Figure3 gives the comparison of the first modal displacement for uncontrolled case and the three controlled cases. It can be seen from the plot that the control is excellent when it is assumed that the mathematical model is the true representation of the actual system (caseI). But when the stiffness matrix is perturbed (case II) or the mass matrix is perturbed (caseIII) then the vibration suppression is poor. There is not much vibration control visible as expected since Kalman filter is based on the original mathematical model but the actual system controlled is considered to be slightly different from it. There is an initial reduction in amplitude of displacement in case III but it again increases after 2 seconds. This is due to mismatch between the actual system and the model used in the control law. Thereafter controller is not effective as the amplitude reduction with time is not significant.

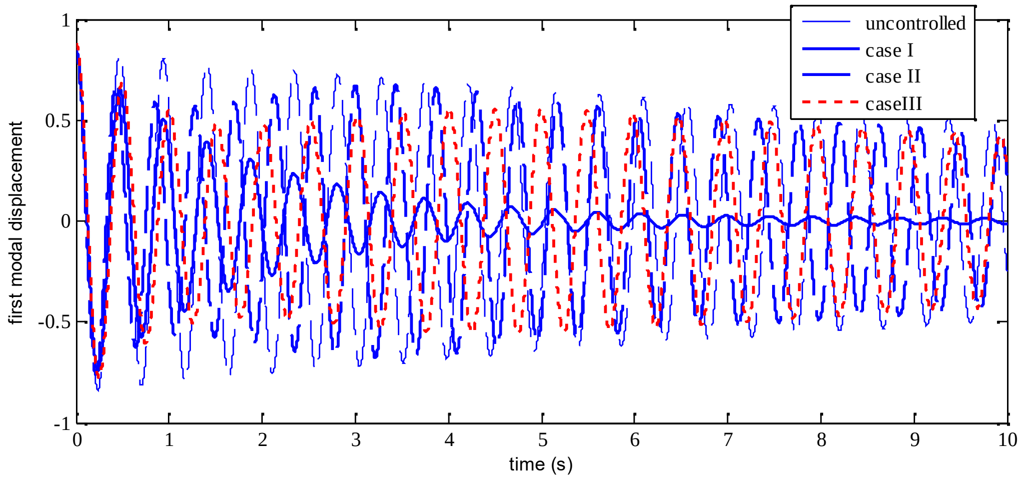

Figure 3: Time response of first modal displacement in the three cases

4 Conclusions

Simulation results for an active vibration control of a four dof system show that the control performance is appreciably affected by the variation in mass and stiffness matrices. When the stiffness matrix and mass matrix are perturbed then the vibration suppression deteriorates drastically. Considerable difference in time response is observed when the Kalman observer based on original mathematical model is used to control the perturbed model representing actual system. Present work suggests that the effectiveness of the controller in suppressing the vibrations of a four dof system is appreciably deteriorated when there is a mismatch between the mathematical model and the actual system.

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