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Study On Behavior of Lateral Torsional Buckling for Beams

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ABSTRACT

The singly symmetric beam subjected to transverse loading applied at different heights on the cross section was conducted for buckling analysis. The beam will tend to bending and twisting. In case of this beam which is symmetric only about minor axis, the elastic critical moment for lateral torsional is calculated. The elastic critical moment may be determined by a buckling analysis of the beam provided that the calculation accounts for all the parameters liable to affect the value of M_{cr} *(critical moment):*

1. *Geometry of the cross-section*
2. *Warping rigidity*
3. *Position of the transverse loading with regard to the shear centre*
4. *Restraint conditions*

Key Words:lateral torsional buckling, critical moment, shear center, warping constant.

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INTRODUCTION

Generally, a beam resists transverse loads by bending action. In a typical building frame, main beams are employed to span between adjacent columns; secondary beams when used – transmit the floor loading on to the main beams. In general, it is necessary to consider only the bending effects in such cases, any torsional loading effects being relatively insignificant.

Buckling instability is a treacherous phenomenon in structural engineering where a small increase in the load can lead to a sudden catastrophic. There are five ways a compression member can buckle or become unstable. Those are:

1. flexural buckling
2. Torsional buckling
3. flexural torsional buckling
4. elastic flexural torsional buckling
5. Lateral torsional buckling

Flexural buckling: This type of buckling can occur in any compression member that experiences a deflection caused by bending or flexure. Flexural buckling occurs about the axis with the largest slenderness ratio, and the smallest radius of gyration.

Torsional buckling: This type of buckling only occurs in compression members that are doubly-symmetric and have very slender cross sectional elements. It is caused by a turning about the longitudinal axis. Torsional buckling occurs mostly in built up sections, and almost never in rolled sections.

Flexural torsional buckling: This type of buckling only occurs in compression members that have unsymmetrical cross section with one axis of symmetry. Flexural torsional buckling and twisting of a member. This mostly occurs in channels, structural tees, double angle shapes, and equal leg single angles.

Lateral buckling of beams has to be accounted for at all stages of construction, to eliminate the possibility of premature collapse of the structure or component. For example, in the construction of steel-concrete composite buildings, steel beams are designed to attain their full moment capacity based on the assumption listed below that the flooring would provide the necessary lateral restraint to the beams. However, during the erection stage of the structure, beams may not receive as much lateral

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support from the floors as they get after the concrete hardens. Hence, at this stage, they are prone to lateral buckling, which has to be consciously prevented.

Two important assumptions have been made therein to achieve this ideal beam behavior. They are:

1. The compression flange of the beam is restrained from moving laterally, and
2. Any form of local buckling is prevented

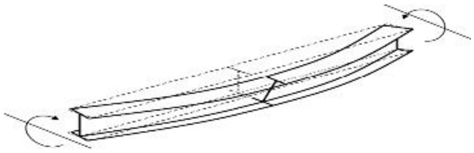

Figure 1beam buckling with moments at supports

1.1. Influence of Cross Sectional Shape On Lateral Buckling:

Structural sections are generally made up of either open or closed sections. Examples of open and closed sections are shown in below figure 2. Cross sections, employed for columns and beams (I and channel), are usually open sections in which material is distributed in the flanges, i.e. away from their centroids, to improve their resistance to in-plane bending stresses. Open sections are also convenient to connect beams to adjacent members. In contrast, closed sections such as tubes, boxes and solid shafts have high torsional stiffness, often as high as 100 times that of an open section. The hollow circular tube is the most efficient shape for torsional resistance, but is rarely employed as a beam element on account of the difficulties encountered in connecting it to the other members and lesser efficiency as a flexural member.

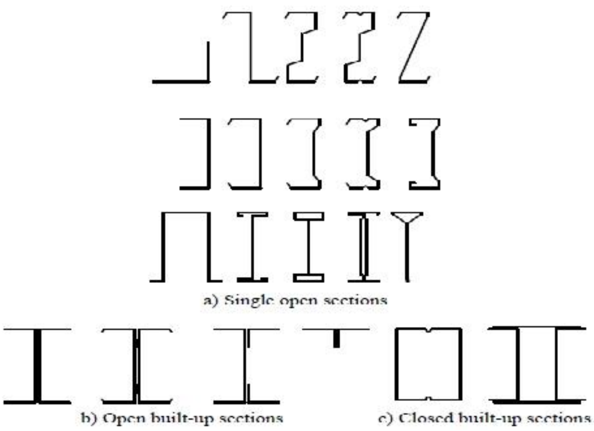

Figure 2 influence of cross section shapes on lateral buckling

1.2. Lateral Torsional Buckling:

Lateral torsional buckling may occur in an unrestrained beam. A beam is considered to be unrestrained when its compression flange is free to displace laterally and rotate. When an applied load causes both lateral displacement and twisting of a member lateral torsional buckling has occurred.

Some factors that influence the lateral torsional buckling behavior of beams are briefly discussed below:

1. Location of the applied load:

The vertical distance between the load application point and the shear center of the section affects the susceptibility of the section to the effects of lateral torsional buckling. If the load is applied at a location above the shear center of a section, it is more susceptible to lateral torsional buckling than if the load was applied through the shear center.

2. The shape of the applied bending moment:

The buckling resistance for a section subject to a uniform bending moment distribution along its length is less than the buckling resistance obtained for the same section subjected to a different bending moment distribution.

3. End support conditions:

Failure modes of beam during buckling are:

1. Laterally stable steel beams can fail only by (a) Flexure (b) Shear or (c) Bearing, assuming the local buckling of slender components does not occur. These three conditions are the criteria for limit state design of steel beams.

2. Flexural failure occurs when the beam fails in bending. Or when the lateral loads on the beam increases beyond its limit then this kind of failure take place. But there is one important failure of beams. Failure due to lateral torsional buckling. This kind of failure happens when compression flange of the beam is not restrained.

1. For short beams the failure will occur by plastic action

2. For Slender unrestrained beams the failure will occur by elastic instability

3. For Intermediate slenderness beams the failure will occur by both plasticand instability were interact

4. The end conditions can significantly influence instability bracing can be used to prevent instability – stiffness and strength is important

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5. Beams will fail by flexural yielding also

1.3. Level of load applied: The level of load applied on a beam is based on three different type of loading. There are shown in the below figure 3

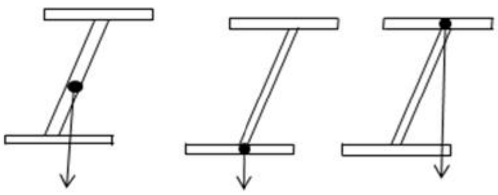

Figure 3(a), 3(b), 3(c) level of load applied

The explanation of the above figure for

Figure 3(a): when the load acting on shear centre of the I-section beam. Then the level of load applied is **'perfect'**.

Figure 3(b): when a load applied on the bottom of the flange. Then the beam **'tends to limit the twisting action'**.

Figure 3(c): when a load applied on the top of the flange. Then the level of load applied tends to **'fail'**.

1.4. Behavior of beam by buckling:

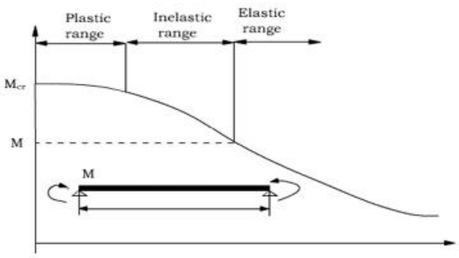

Figure 4 behavior of beam by buckling

The above figure (4) is the buckling behavior of beam in between critical moment (M_{cr}) and end moment of the beam (M). There are three types of ranges according to buckling. There are: Plastic range, Inelastic range, Elastic range

This behavior can be classified under two parts:

a. When the beam is adequately supported against lateral buckling, the beamfailure occurs by yielding of the material at the point of maximum moment. The beam is thus capable of reaching its plastic moment capacity under the applied loads. Thus thedesign strength is governed by yield stress and the beam is classified as laterallysupported beam.

b. Beams have much greater strength and stiffness while bending about the major axis. Unless they are braced against lateral deflection and twisting, they are vulnerable to failure by lateral torsional buckling prior to the attainment of their full in plane plastic moment capacity. Such beams are classified as laterally supported beam.

2. LITERATURE STUDY

Codes related to lateral torsional buckling are:

1. IS 800-2007:general construction in steel
2. AISC 360-2010:specification for structural steel buildings
3. BS 5950:design procedure for steel frame structures

2.1. Derivation of critical elastic moment (M_{cr}):

Taking unlike circular and I-section respond to torsional$M_{torsion}$= $M_{st.venant}$ + $M_{warping}$ ---------------**(2.1)**

Pure torsion or st. venant:

When we apply torsion to section it will respond by rotating angle Φ

$$\frac{\mathbf{Ms}\,.\,\mathbf{v}}{G} = \frac{d}{d}$$

Therefore $\text{Mst. venant} = \text{GJ}\frac{d}{d}$ -------**(2.2)**

no warpingin web

Focus all of our attention in forces in flanges. Relate shear force in flange to warping moment

Displacement on flange $u_f = \frac{h}{2}$ P

Third degree: $[\frac{du_f}{d}]^3 = \frac{h}{2}[\frac{d}{d}]^3$--------**(2.3)**

We can use this to determine bending moment in flange. Relation between shear force and moment

$\frac{dM_f}{d} = v_f$

For mechanics $[\frac{du_f}{d}]^2 = -\frac{M_f}{EI_f}$

Therefore $[\frac{du_f}{d}]^3 = -\frac{v_f}{EI_f}$------------ **(2.4)**

Setting (2.1) & (2.4) we get

$M_{warping}$ = (Shear) $V_f h = -EI_f \frac{h}{2}[\frac{d}{d}]^3 h$

Therefore, $M_{warping} = -E\,\frac{I_f.h.h}{2}[\frac{d}{d}]^3$

Where C_w (warping constant) = $I_y.[\frac{h}{2}]^2$

$M_{warping} = [-E\,C_w[\frac{d}{d}]^3]$ ------- **(2.5)**

Now substitute (2.1) & (2.5) in equation (1), we get

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$M_{torsion} = GJ\frac{d}{d} - EC_w[\frac{d}{d}]^3 = (GJ)\Phi' - (EC_w)\Phi'''$-- **(2.6)**

2.2. For calculating M_{cr}:

For deriving, consider a beam subjected to uniform moment and prevent warping. There are 3 moments considered in the beam:

- Vertical deflection(yz plane)
- Lateral deflection(xz plane)
- Twisting moment (along z-axis)

1. Yz-plane: it serves as a reminder of general beam deflection theory

$M_o = -EI_x [\frac{d}{d}]^2$

Therefore, $M_o = -[EI_x]\, v''$------- (2.7)

2. Xz-plane: $M_o \sin(\Phi) = -EI_y[\frac{d}{d}]^2$

$\sin(\Phi) \sim \Phi$

$M_o\Phi = -(EI_y)u''$------ (2.8)

3. Along z-axis: i.e. twisting moment or torsional moment which is obtained above eq(c)

$M_{torsion} = (GJ)\,\Phi' - (EC_w)\,\Phi'''$

The slope of the beam (θ) and lateral deflection can be related:

$M_z = -M_o\theta = -M_o[\frac{d}{d}]$

$M_z = -(M_o)\,u'$--------- (2.9)

Then setting (2.8) & (2.9) we get

$-(M_o)u' = (GJ)\Phi' - (EC_w)\Phi'''$

Again derivating the above equation, we get

$-(M_o)u'' = (GJ)\Phi'' - (EC_w)\,\Phi''''$ ----- (2.10)

From eq (2.1)

$u'' = -\frac{M}{E}\Phi$

Substituting u'' value in eq(2.4), we get

$(EC_w)\Phi'''' - (GJ)\Phi'' - \frac{M^2}{E}\Phi = 0$--------(2.11)

The above equation fourth order polynomial equation Assume deflection and moment at each end of unbraced length equal to zero as per twisting moment. Columns have solution in sinusoidal

$\Phi(z) = k\sin[\frac{\pi}{L_b}]$------------------------------- (2.12)

Where $\sin\pi = 0$

Where k = twist at mid span, L_b = unbraced length

$\Phi'(z) = [\frac{k}{L_b}]\cos[\frac{\pi}{L_b}]$---------------------- (2.13)

$\Phi''(z) = -[\frac{k\pi^2}{L_b{}^2}]\sin[\frac{\pi}{L_b}]$------------------- (2.14)

$\Phi'''(z) = -[\frac{k\pi^3}{L_b{}^3}]\cos[\frac{\pi}{L_b}]$------------------- (2.15)

$\Phi''''(z) = [\frac{k\pi^4}{L_b{}^4}]\sin[\frac{\pi}{L_b}]$-------------------(2.16)

Substitute above values in eq (2.11) i.e. fourth order equation

$EC_w[[\frac{k\pi^4}{L_b{}^4}] \sin[\frac{\pi z}{Lb}] - GJ(-[\frac{k\pi^2}{L_b{}^2}]\sin[\frac{\pi z}{Lb}]) + [\frac{M^2}{E}]\, k \sin[\frac{\pi z}{Lb}]] = 0$

$M_o = [\frac{\pi}{Lb}]\sqrt{EI\cdot GJ - [\frac{\pi E}{Lb}]^2 I\cdot C}$

$EI_yGJ - [\frac{\pi E}{Lb}]^2$ The elastic buckling moment M_{cr}, according to lateral torsional buckling is

$M_{cr} = M_o = [\frac{\pi}{Lb}]\sqrt{EI\cdot GJ - [\frac{\pi E}{Lb}]^2 I\cdot C}$ ------ (2.17)

- According to deflection theory

$M_{cr} = [\frac{\pi}{L}]\sqrt{EI\cdot GJ}$-------- (2.18)

Where EI_y = minor axis flexure rigidity, GJ = torsional rigidity, L = length of the beam = 1

1. For plunge beam: plastic moment is less in critical moment and stability does not occur, no slender.

2. For slender beam: beam will fail by lasting stability and simple to analyze

3. For medium slender beam: plastic &stability inter hacked

- For instability moment (prevent warping)

$M_{cr} = [\frac{\pi}{L}]\sqrt{EI\cdot GJ} * \sqrt{1 + \frac{\pi 2E\,Iw}{L^2 GJ}}$------- (2.19)

M_{cr} for different beam sections, considering loading, support condition and non-symmetric section on minor axis according to IS:800-2007, shall more accurately calculated by using following equation:

$M_{cr} = [C_1 \left[\frac{\pi^2 EI_y}{L_L}\right]\{[\left[\frac{k}{k_w}\right]^2 \frac{I_w}{I_y} + \frac{GI_tL_L{}^2}{\pi^2 EI_y} + (C_2\, y_g - C_3 y_j)^2]^{0.5} - (C_2\, y_g - C_3 y_j)\}$(2.20)

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Where,c1, c2,c3=factors depending upon the loading and end restrained conditions, I_t= torsional constant for open section, I_w= warping constant, I_y,r_y= moment of inertia, radius of gyration about the weak axis, respectively, L_{LT}= effective length for lateral torsional buckling, h_f= Center to center distance between flanges, t_f= thickness of the flange, G = Modulus of rigidity, K= effective length factors of the unsupported length accounting for boundary conditions at the end lateral supports. 1.0 for free rotate about weak axis, K_w= warping restraint Factor. Kw should be taken as 1.0., Y_g,y_j =y distance between the point of application of the load and the shear centre of the cross section

3. Results and Discussions

Example for How I Solve Problem and I Get Above Values ISLB 100:

h= 100mm, B= 50mm, t_w= 4mm, t_f= 6.4mm, I_y= 12.7*10^4mm^4, d= 73mm, r_y= 11.2mm, f_y= 250N/mm^2, E=2×10^5N/mm^2,L=6000mm

b= $\frac{B}{2}$= $\frac{5}{2}$= 25mm

$\frac{b}{t_f}$= $\frac{2}{6.4}$= 3.90 < 9.4ε ; ε= $\sqrt{\frac{2}{f_y}}$,$\frac{d}{t_w}$= $\frac{7}{4}$=18.25<84 ε

As $\frac{b}{t_f}$<9.4 &$\frac{d}{t_w}$<84 the section is plastic section as per IS 800:2007. equation (2.20)

M_{cr}=$[C_1\left[\frac{\pi^2 EI_y}{L_L}\right]\{\left[\frac{k}{k_w}\right]^2\frac{I_w}{I_y}+\frac{GI_tL_L{}^2}{\pi^2 EI_y}+(C_2\,y_g-C_3y_j)^2]^{0.5}-(C_2\,y_g-C_3y_j)\}$(eq (2.20))

C_1=1.283, C_2=0, C_3=0.993, k=k_w=1,
G=0.769×10^5N/mm^2, L_{LT}=1.0L=6000mm,
y_g=0.5h=0.5×0.10=0.05mm,

Y_j=$\frac{1(2\beta_f-1)\,h_y}{2}$; where β_f=0.5, h_y= h- t_f =100-6.4= 93.6mm then y_j=0 and y_s=0

I_t= $\frac{\Sigma biti^2}{3}$= 2×50×6.4^3+ (100-2×6.4)×4^3= 317.9×10^4mm^4

I_w= (1- β_f) $\beta_f I_y h_y{}^2$= (1-0.5) ×0.5×12.7×10^4× (93.6)^2= 27816.048×10^4mm^4

M_{cr}=1.283× $\frac{\pi^2\times 2\times 1\ \ {}^{9}\times 1\ \ .7}{6\ \ \ {}^{2}}$ × {($\frac{2\ \ \ \ .0\ \ \times 1\ \ {}^{4}}{1\ \ .7\times 1\ \ {}^{4}}$} + $\frac{0.7\ \ \times 1\ \ {}^{5}\times 3\ \ \ .9\times 1\ \ {}^{4}\times(6\ \ \ \ )^2}{\pi^2\times 2\times 1\ \ .7\times 1\ \ {}^{9}}$}$^{0.5}$=893424[2190.24+35106368.84]$^{0.5}$= 5293758379= 5293758.37×10^4N-mm

I have done all steel sections using steel tables and IS: 800 2007. The values I got below are:

	b/tf (mm)	d/tw (mm)	Mcr(N-mm)		b/tf (mm)	d/tw (mm)	Mcr(N-mm)
ISLB 100	3.9	18.2	529375888	ISMB100	5.20	16.2	131937906
ISLB 125	5.76	21.6	123635831	ISMB 125	4.93	20.2	151016503
ISLB 150	5.88	24.3	318855783	ISMB 150	5.26	23.7	178262060
ISLB 175	6.52	27.7	216098257	ISMB 175	5.23	24.4	912444620
ISLB 200	6.84	30.6	292851265	ISMB 200	4.62	26.7	544790072
ISLB 225	5.81	31.0	362341747	ISMB 225	4.66	26.6	811342106
ISLB 250	7.62	33.2	506164199	ISMB 250	5	28.1	117614293
ISLB 300	7.97	36.5	936049793	ISMB 300	5.64	32.2	143089917
ISLB 350	7.23	38.9	165442862	ISMB 350	4.92	35.5	200826894
ISLB 400	6.6	42.0	204182929	ISMB 400	4.37	37.5	260350303

Table 1 manual calculations

For channel sections values are:

Table 2 for channel sections

	b/t_f(mm)	d/t_w(mm)	M_{cr} (N-mm)		b/t_f(mm)	d/t_w(mm)	M_{cr} (N-mm)
ISLC 100	3.90	18.5	74318302	ISMC 100	3.33	13.6	96131234
ISLC 125	4.92	21.9	13837485	ISMC 125	4.01	17.0	18703194
ISLC 150	4.80	24.3	25356202	ISMC 150	4.16	19.7	30814000
ISLC 175	3.94	27.1	36911469	ISMC 175	3.67	22.52	40328648
ISLC 200	3.47	29.0	48212159	ISMC 200	3.28	24.62	51417802
ISLC 225	4.41	30.3	60066919	ISMC 225	3.22	26.70	69575800
ISLC 250	4.67	32.6	82328286	ISMC 250	2.83	27.11	91424246
ISLC 300	4.31	36.6	10408793	ISMC 300	3.30	31.67	11545644
ISLC 350	4	39.4	12842547	ISMC 350	3.70	35.56	140932299
ISLC 400	3.57	42.1	16587374	ISMC 400	3.26	38.69	120564336

GRAPHS ARE OCCURRED BY USING TABLE 1 AS SHOWN BELOW:

There are four classes to define the cross sections. There are:

1. Class 1 (plastic): cross sections which can develop plastic hinges and have rotation capacityrequired for failure of the structure of plastic mechanism. Most of the sections i.e. ISLB, ISMB, ISLC, ISMC etc. are come under this category according to my project values.

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2. Class 2 (compact): cross section which can develop plastic moment of resistance, but have inadequate plastic hinge rotation capacity for formation of plastic mechanism, due to local buckling.

3. Class 3 (semi- compact): cross sections in which extreme fibre in compression can reach yield stress, but cannot develop the plastic moment of resistance, due to local buckling. Section ISHB comes under this category according to my project values.

4. Class 4 (slender): cross sections in which the elements buckle locally even before reaching yield stress.

The graph is drawn between $\frac{b}{t_f}$, $\frac{d}{t_w}$ & M_{cr}. The graph is based on the above calculations. There are some graphs shown below:

FOR ISLB:

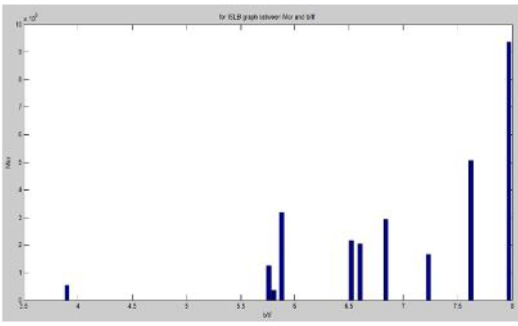

Figure 6(a) ISLB Mcr vs. b/t_f

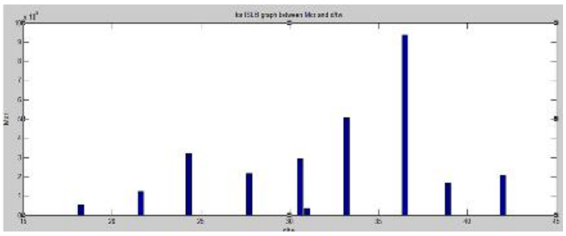

Figure 6(b) ISLB Mcr vs. d/tw

FOR ISMB:

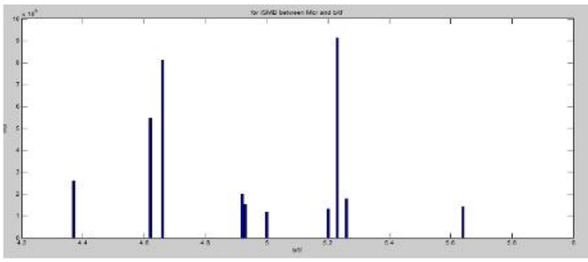

Figure 6(c) ISMB Mcr vs. b/tf

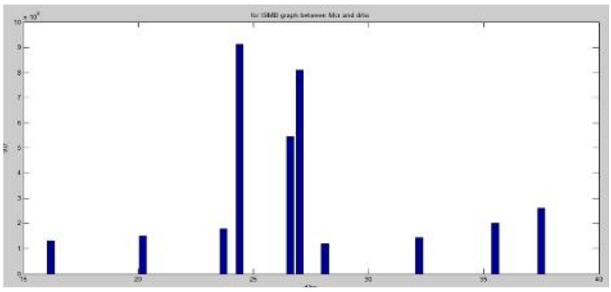

Figure 6(d) ISMB Mcr vs. d/tw

FOR ISLC:

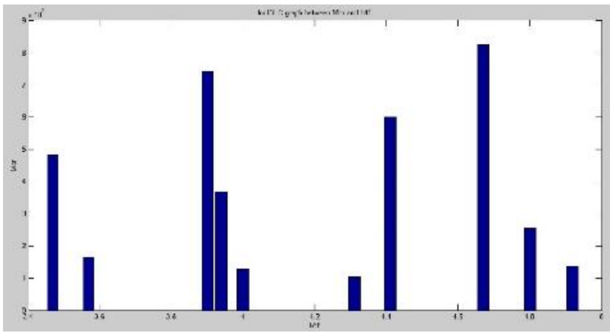

Figure 6(e) ISLC Mcr vs. b/tf

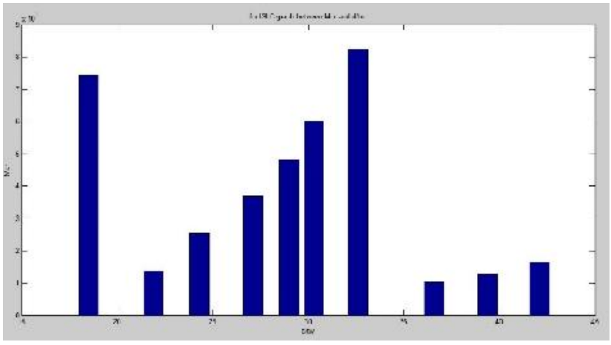

Figure 6(f) ISLC Mcr vs. d/tw

FOR ISMC:

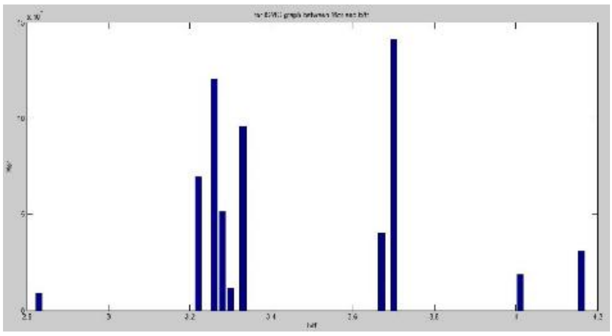

Figure 6(g) ISMC Mcr vs. b/tf

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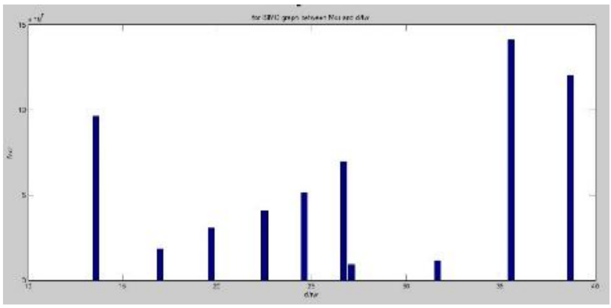

Figure 6(h) ISMC Mcr vs. d/tw

4. Adequate Against Lateral Torsional Buckling for the Applied Bending Moment for A Beam

After calculating M_{cr} value the beam is calculated against lateral torsional buckling for load is applied bending moment. Example is given below:

Calculation of f_{bd}:For ISMB 450- h= 450mm, B=150mm, t_f = 17.4mm, t_w = 9.4mm, I_y =834χ10^4mm^4, r_y = 30.1mm, d= 379.2mm, M_{max} = (end moments) = 202 kN- m, G= 0.769χ10^5 N/mm^2, C_1 = 1.283, C_2 = 0, C_3 = 0.993, L=6m, E= 2χ10^5 N/mm^2,M_{cr} = 357169.20χ10^3 N-mm, γ_m =0.10, f_y = 250N/mm^2

(Design bending stress)$f_{bd} = \chi \frac{f_y}{\gamma_m} <= \frac{f_y}{\gamma_m}$, $\lambda = \sqrt{\frac{\beta_b z_p f_y}{M_c}}$

Where β_b = 1 for plastic and compact section, z_p = plastic/elastic section moduli of cross section= 1533.36χ10^3mm^3, f_y = yield stress of material =250N/mm^2

$\lambda = \sqrt{\frac{1\chi \quad .3 \quad 1 \ ^3\chi}{3 \quad .2 \ \chi 1 \ ^3}}$ =1. 112

Φ = 0.5[1+α (λ-0.2)+λ^2]

α= 0.21 for rolled steel sections

= 0.49 for welded steel sections

Φ = 0.5[1+0.21(1.12-0.2) + (1. 12)^2] = 1.213

$\chi = \frac{1}{\phi+[\phi^2-\lambda^2]^{0.5}} = \frac{1}{1.2 \quad +[1.2 \quad ^2-1.1 \quad ^2]^{0.5}}$ = 0.589

f_{bd} = 0.589 χ250/ 1. 10 = 133.92N/mm^2

Hence design bending strength M_d = β_b z_p f_{bd} = 1 χ1533.36χ10^3χ133.92 =205.34kN-m

M_d> M_{max}

205.34>202

Therefore, ISMB 450 is adequate against lateral torsional buckling for assumed applied bending moments.

5. MOMENT FACTOR (C_1, C_2, C_3)

5.1.The equivalent uniform moment factor, C1: In literature, the C1–factor is referred to as the equivalent uniform moment factor or the moment gradient factor. The critical buckling load can be derived for different load cases through methods of potential energy. Examples are given below for a double symmetric beam on fork supports, subjected to a concentrated load and a uniformly distributed load (Yoo & Lee 2011).

1. Critical load for a concentrated load in mid-span, fork supports

$P_{cr} = \frac{4.\pi^3}{L^3}\sqrt{\frac{3}{\pi^2+6} EI_z(GI_t + \frac{EI_w.\pi^2}{L^2}}$ [N] -- (5.2)

2. Critical load for a uniformly distributed load, fork supports

$q_{cr} = \frac{2.\pi^3}{L^3}\sqrt{\frac{3}{\pi^4+4} EI_z(GI_t + \frac{EI_w.\pi^2}{L^2}}[\frac{N}{m}]$--- (5.3)

When equation (4.2) and (4.3) are rewritten as expressions of the elastic critical moment, a comparison can be made with Mcras reference. The influence of the

Different loading conditions can now be studied.

1. Critical load for a concentrated load in mid-span, fork supports

$M_{cr} = \frac{Pc \ .L}{4} = 1.36 \frac{\pi^2.E\,I_z}{L^2}\sqrt{\frac{I_w}{I_z} + \frac{L^2.GI_t}{\pi^2.EI_z}}$ [Nm] ---- (5.4)

2. Critical load for a uniformly distributed load, fork supports

$M_{cr} = \frac{qc \ .L^2}{8} = 1.13 \frac{\pi^2.E\,I_z}{L^2}\sqrt{\frac{I_w}{I_z} + \frac{L^2.GI_t}{\pi^2.EI_z}}$ [Nm] ----(5.5)

If equation (4.4) and (4.5) are compared to the reference equation (4. 1), the only difference is a factor 1.36 and 1.13 respectively. Hence, it is expected that the elastic critical moment for a general load case can be written as:M_{cr} = $C_1\frac{\pi^2.E\,I_z}{L^2}\sqrt{\frac{I_w}{I_z} + \frac{L^2.GI_t}{\pi^2.EI_z}}$--- (5.6)

Where C1 take the following values

- A concentrated load in mid-span, fork supports, C1 = 1.36
- A uniformly distributed load, fork supports, C1 = 1.13
- Reference load case, Mcr, C1 = 1.0

In cases where table values are not available, C1 can be calculated with closed-form expressions. By using curve fitting-techniques, several researchers

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have presented approximate expressions of C1, generally constructed to give lower-bound solutions. Exact expressions are difficult to construct since C1 is dependent on more factors than the shape of the moment diagram. There are some expressions given below:

1. Closed-form expression by M.G. Salvadori:

$C_1 = 1.75- 1.05\Psi+ 0.3\Psi^2 <= 2.3$

Where Ψ = ratio of end-moments, $\frac{M_1}{M_2}$

2. Closed-form expression by P.A. Kirby and D.A. Nethercot:

$C_1 = \frac{1 \quad M_m}{3M_A+4M_B+3M_C+2M_m}$

Where M_{max} = Maximum moment, M_A, M_B, M_c = absolute values of the moments at L/4,L/2,3L/4 respectively.

1. Closed form expression by Serna et al. (2005) and Lopez et al. (2006)

$C_1 = \frac{\sqrt{\sqrt{k.A_1}+(\frac{1-\sqrt{k}}{2}A_2)^2}+(\frac{1-\sqrt{k}}{2})A_2}{A_1}$

$K=\sqrt{k_z.k_w}$

$A_1 = \frac{M \quad {}^2+\alpha_1.k.M1^2+\alpha_2.k.M2^2+\alpha_3.k.M3^2+\alpha_4.k.M4^2+\alpha_5.k.M5^2}{(1+\alpha_1+\alpha_2+\alpha_3+\alpha_4+\alpha_5)M \quad {}^2}$

$A_2 = \left|\frac{M_1+2M_2+3M_3+2M_4+M_5}{9M_m}\right|$

Where $\alpha_1 = (1-k_W)$, $\alpha_2 = 5\frac{k \quad ^2}{k \quad ^2}$, $\alpha_3 = 5\{\frac{1}{k_z}+\frac{1}{k_w}\}$, $\alpha_4 = 5\frac{k \quad ^3}{k \quad ^2}$, $\alpha_5 = (1-k_z)$, k_z = effective length factor related to bending at the boundaries, k_w = effective length factor related to warping at the boundaries

5.2.Correction factor for the point of load application, *C2*

Equation (5), with the *C1*-factor added, is only valid when the load acts in the shear Centre. In reality, beams are often loaded on the top or bottom flange, and not in the shear Centre (*SC).* Thus a second correction factor, *C2*, has been added to account for the effects of the point of load application, (PLA). With the *C2*-factor added to the formula, equation below is obtained.

$M_{cr} = [C_1\left[\frac{\pi^2 EI_y}{L_L}\right]\{\left[\frac{k}{k_w}\right]^2\frac{I_w}{I_y}+\frac{GI_tL_L \quad ^2}{\pi^2 EI_y}+(C_2\, z_g)^2]^{0.5}-(C_2\, z_g-C_3 z_j)\}$ ----------(eq(2.20))

Where *C2* = Correction factor for the point of load application

zg = Distance between the point of load application and the shear Centre, measured positive above *SC.*

5.3.Correction factor for cross-section asymmetry, C3

With the C1- and C2-factors, the 3-factor formula can be used for calculation of Mcr for double-symmetric beams with various loads and points of load application. Still,the equation is only valid for double-symmetric sections, i.e. sections where both themajor and minor axis are symmetry lines of the section. In order to analyticallyestimate Mcr for sections with symmetry only about the minor axis, a third correctionfactor C3 must be introduced.With *C3* added, the expression takes the following form. i.e.

$M_{cr} = [C_1\left[\frac{\pi^2 EI_y}{L_L}\right]\{\left[\frac{k}{k_w}\right]^2\frac{I_w}{I_y}+\frac{GI_tL_L \quad ^2}{\pi^2 EI_y}+(C_2\, z_g)^2]^{0.5}-(C_2\, z_g-C_3 z_j)\}$ --------(eq(2.20))

Where *C3* = Correction factor for asymmetry about *y*-axis, z_j = distance related to the effects of asymmetry about y-axis. *Zs* = Distance between the shear centre and the centre of gravity. For a more detailed definition, see ECCS (2006).

The *C3*-factor is not studied further since the thesis is limited to double-symmetric sections.

6. ACCORDING TO VARIOUS CODE BOOKS C_1, C_2, C_3

6.1. Moment line with maximum at the start or at the end of the beam

Table 3 moment line with maximum at the end of the beam

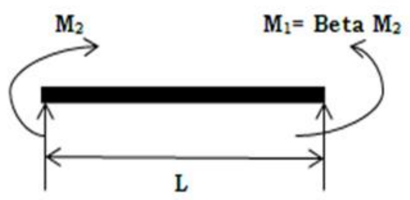

Figure 9 Moment line with maximum at the start or at the end of the beam

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6.2. Moment distribution generated by F load

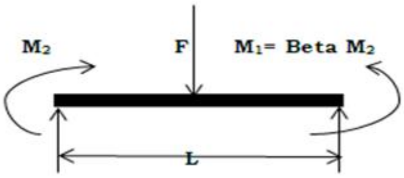

Figure 10 Moment distribution generated by F load

Table 4 Moment distribution generated by F load

Codes	C1	C2	C3
DIN18800	1.77-0.77β	0	-
ONORM 4300	1.77-0.77β	0	-
ENV 1993	$1.88-1.40\beta+0.52\beta^2$	0	-
IS 800	$1.88-1.40\beta+0.52\beta^2$	0	-
CM66	$\frac{\sqrt{3}}{1+\beta+\beta 2+0.152(1-\beta)2}$	0	-
NEN 6770/6771	$1.75+1.05\beta+0.30\beta^2$	0	-
SIA 263	$1.75+1.05\beta+0.30\beta^2$	0	-

	C1	C2	C3
If $M_2>0$	A**(2.75B**+1)1.35+B**(-1.62A**+1)E**	$0.55A^{**}(1+C^{**}e^{D^{**}}(1/2\beta+1/2))$	-
If $M_2<0$	$1.35A^{**}+B^{**}E^{**}$	$0.55A^{**}$	-

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6.3.Moment distribution generated by q load

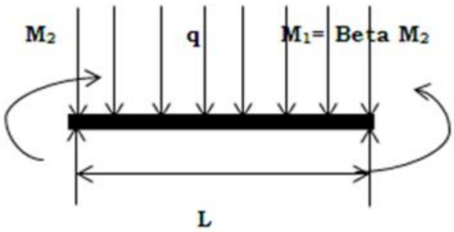


Figure 11 Moment distribution generated by q load

Table 5 Moment distribution generated by q load

Codes		C1	C2	C3
1. ENV 1993, IS 800, CM66 2. DIN 18800 3.ENV 1993&IS 800	If $M_2>0$	1.13A**+B*E*A*(1.45β*+1)1.13+B*(-0.71A*+1)E* 1.12A*+B*E*A*(1.45B*+1)1.12+B*(-0.71A*+1)E* 1.88-1.40β+0.52β^2	0.75A*	-
	If $M_2<0$		0.45A*(1+C*e^D*(1/2β+1/2))	
	If $M_2>0$		0.45A*	
	If $M_2<0$		0.45A*(1+C*e^D*(1/2β+1/2))	
			-	
4. NEN 6770/6771		1.75-1.05β+0.30β^2	-	-
CM66		1.88-1.40β+0.52β^2	-	-

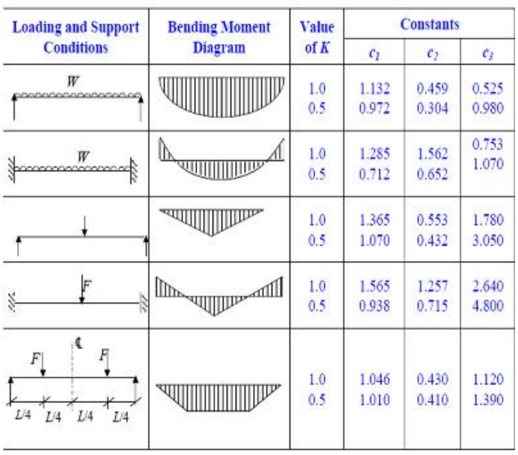
Where,

A*=ql^2/8[M2] +ql^2; C*=94[M2]/ql^2; B*=8[M2]/8[M2] +ql^2; D*=-72{[M2]/ql^2}^{2}

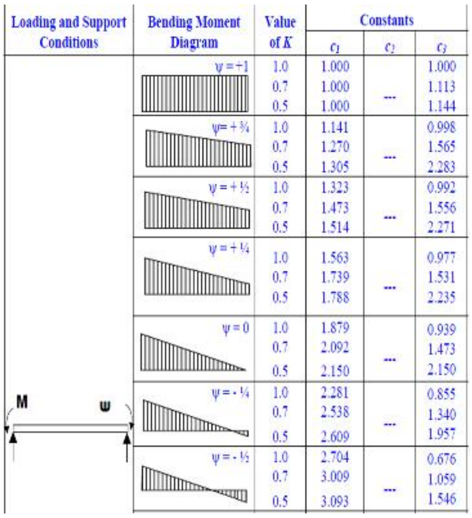
A**=F1/4[M2]+F1; B**=4[M2]/(4[M2]+F1); C**=38[M2]/F1; D**=-32{[M2]/F1}^{2}

6.4. From IS 800:2007 (clause E- 1.2)

Table 6 from IS 800: 2007



Loading and Support Conditions	Bending Moment Diagram	Value of K	Constants c_1	c_2	c_3
W		1.0 0.5	1.132 0.972	0.459 0.304	0.525 0.980
W		1.0 0.5	1.285 0.712	1.562 0.652	0.753 1.070
		1.0 0.5	1.365 1.070	0.553 0.432	1.780 3.050
F		1.0 0.5	1.565 0.938	1.257 0.715	2.640 4.800
F F L/4 L/4 L/4 L/4		1.0 0.5	1.046 1.010	0.430 0.410	1.120 1.390



Loading and Support Conditions	Bending Moment Diagram	Value of K	Constants c_1	c_2	c_3
M ψ	ψ = +1	1.0 0.7 0.5	1.000 1.000 1.000	---	1.000 1.113 1.144
	ψ= + ¾	1.0 0.7 0.5	1.141 1.270 1.305	---	0.998 1.565 2.283
	ψ = + ½	1.0 0.7 0.5	1.323 1.473 1.514	---	0.992 1.556 2.271
	ψ = + ¼	1.0 0.7 0.5	1.563 1.739 1.788	---	0.977 1.531 2.235
	ψ = 0	1.0 0.7 0.5	1.879 2.092 2.150	---	0.939 1.473 2.150
	ψ = - ¼	1.0 0.7 0.5	2.281 2.538 2.609	---	0.855 1.340 1.957
	ψ = - ½	1.0 0.7 0.5	2.704 3.009 3.093	---	0.676 1.059 1.546

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7. C_1, C_2, C_3 MANUAL VALUES COMPARING TO IS 800 VALUES

The manual calculations I done is based on previous chapter equations for C_1, C_2, C_3. I created a table by calculating the C_1, C_2, and $C_3$for one section i.e. ISLB 400 as shown below. For ISLB 400 data is:

h= 400mm, B=165mm, t_f = 12.5mm, t_w = 8mm, I_y = 716.4*10^4 mm^4, r_y = 31.5mm, d= 336.2mm, M_{cr} = 204129.29*10^3 N-mm, I_w = 26892984.38*10^4 mm^4, I_t = 8365.31*10^4 mm^4, G = 0.769*10$^5 \frac{N}{mm^2}$

1. C_1Table 6 showing c_1 values with various code equations

	Equation		**C_1 value**	**Absolute value as per IS 800: 2007**
IS 800: 2007	**Mcr** $= C_1 \frac{\pi^2.EI_y}{L^2}\sqrt{\frac{I_w}{I_y}+\frac{G.I_t.L^2}{\pi^2.E.I_y}}$		1.283	1.283
M.G. Salvadori	**C_1** = 1.75-1.05ψ+0.30ψ2	Ψ = 1	1	1
		Ψ = 0.75	1.13	1.14
		Ψ = 0.50	1.3	1.323
		Ψ = 0.25	1.506	1.563
		Ψ = 0	1.75	1.879
DIN18800, ONORM 4300	**C_1=** 1.77-0.77β	β = 0.5	1.385	
ENV 1993, IS 800	**C_1** = 1.88- 1.40β+0.52β^2	β = 0.5	1.31	
CM66	**C_1** = $\frac{\sqrt{3}}{1+\beta+\beta^2+0.1 \quad (1-\beta)^2}$	β = 0.5	0.968	
NEN 6770/6771, SIA 263	**C_1** = 1.75+ 1.05β+0.30β^2	β = 0.5	2.35	

C_2: C_2 equation is given below:

$M_{cr} = C_1.\frac{\pi^2.E.I_y}{L^2}\left[\sqrt{\frac{I_w}{I_y}+\frac{L^2.G.I_t}{\pi^2.E.I_y}+\left(C_2.y_g\right)^2}\right]-$ $\left(C_2.y_g\right)$-- (6.1)

Always C_2 shows zero for simply supported beam

1. **C_3:** C_3 not mentioned in any code book except IS 800: 2007 but this code consists only c_3 values without equations and only equation given below:

$M_{cr} = C_1.$ $\frac{\pi^2.E.I_y}{L^2}\left[\sqrt{\frac{I_w}{I_y}+\frac{L^2.G.I_t}{\pi^2.E.I_y}+\left(C_2.y_g-C_3.y_j\right)^2}\right]-$ $(C_2.y_g - C_3.y_j)$ ------- (eq(2.20))

Where y_g = 0.5h, h = height of the section

$Y_j = \frac{1\,(2\beta_f - 1)h_y}{2}$

Where β_f = 0.5, h_y =h-t_f

And by using this formula C_3 can't be expand easily and it is complicated.

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