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Design and Parametric Optimization of an Effective Dynamic Vibration Absorber for Reduction in Vibration Amplitude

***Mihir Kumar Sutar, Krishna Chandra Patra**
Department of Mechanical Engineering, Veer SurendraSai University of Technology Burla

ABSTRACT:
Control of excessive vibration is the need of the hour, with its root in many sectors like aerospace (e.g for tracking and pointing), and flexible space structures. Over the past decades concern has also been focused on the civil and infrastructure related issues, such as protection of buildings and bridges from natural extreme loads of earthquakes and wind.With the increase in infrastructure and civil society, there is a sequel need of tall buildings around the world.These buildings are subjected to excessive vibrations. The vibration control systems applied in machines construction can be categorized as follows groups-passive, active and semi-active. Passive vibration control systems can be depicted as systems characterized by dissipation of vibration energy without increasing total energy in primary effects in most of the mechanical systems. Contrary to the active vibration control systems, to the passive ones act on structure by means of actuators generating forces depending on the dynamic response from the structure. Semi-active vibration control system connects passive tunable devices and active systems characterized by control abilities. A simply supported beam with heavy loads mounted along the beam can be used as a basic representative model for a number of advanced flexible engineering structures. This paper presents the methodology adopted for determination of theoretical natural frequencies and mode shape of the simply supported beam type main vibrating system, used in many areas like bridges and double-beam structures. Dynamic vibration absorber (DVA) is designed and developed at one of the resonant frequencies of the main vibrating system. Natural frequencies and mode shapes of main vibrating system obtained theoretically and also by using Finite Element Analysis (FEA). The parameters are then optimized to reduce the vibration amplitude. The performance of the damped and un-damped vibration absorber is then analyzed using ANSYS.

KEYWORDS:Dynamic Vibration Absorber, Modal Analysis, Tuned damped DVA

INTRODUCTION:

Vibration control is having its roots primarily in aerospace related problems such as tracking and pointing, and in flexible space structures.The effects of these vibrations are excessive variable stress in machine componentsundesirable noise looseness of parts and failure of thesystem. Vibrations can cause adverse effects, and they can also be controlled in various ways. The vibration control systems applied in machines construction can be categorized as follows groups-passive, active and semi-active.In order to quench the vibrations a properly designed auxiliary system called adynamic vibrations absorber is coupled to a mainsystem so that motion from the main system istransferred to the auxiliary system hence protectingmain system from harmful effect of vibration.Different types of dynamic vibration absorbers are used to reduce the vibration of single degrees offreedom main system. Design of the vibration absorber plays an important role and so its effectiveness. Many researchers around the globe have put their effort for a properly designed absorber with better effectiveness.

The conventional DVA, with tuned mass damper (TMD) was first proposed by Frahm (1909) and is still quite widely used in practice because of its simplicity. DVA can be used as an effective vibration control device. A theory for the TMD was presented later by Ormondroyd and Den Hartog (1928),followed by a detailed discussion of optimal tuning and damping parameters byDen Hartog (1947). The initial theory was applicable toFrahm'sundampedsingle degree of freedom (SDOF) system subjected to a sinusoidal force excitation.Brock (1946) took a different approach. Based on the results, he suggested the optimum damping ratio can be given for model. Hrovat et al. (1982) presented semi-active tuned mass damper(SATMD), a TMD with time varying controllable damping.Under identical conditions, the behavior of a structure equipped with SATMD instead of TMD is significantly improved. The control design of SATMD is less dependent on related parameters (e.g., mass ratios, frequency ratios and so on), so that there greater choices in selecting them.The first mode

response of a structure with TMD tuned to the fundamental frequency of the structure can be substantially reduced but, in general, the higher modal responses may only be marginally suppressed or even amplified. To overcome the frequency-related limitations of TMDs, more than one TMD in a given structure, each tuned to a different dominant frequency, can be used.

The concept of multiple tuned mass dampers (MTMDs) together with an optimization procedure was proposed by Clark (1988). Since, then, a number of studies have been conducted on the behavior of MTMDs. A doubly tuned mass damper (DTMD), consisting of two masses connected in series to the structure was proposed by Setareh (1992). In this case, two different loading conditions were considered: harmonic excitation and zero-mean white-noise random excitation, and the efficiency of DTMDs on response reduction was evaluated. Analytical results show that DTMDs are more efficient than the conventional single mass TMDs over the whole range of total mass ratios, but are only slightly more efficient than TMDs over the practical range of mass ratios (0.01-0.05).A method of estimating the parameters of tuned mass dampers for seismic applications has been reported by Fahim and Bijan (1997). The authors have presented the optimum parameters of TMD that result in considerable reduction in the response of structures to seismic loading. The criterion that has been used to obtain the optimum parameters is to select for a given mass ratio, the frequency and damping ratios that would result in equal and large modal damping in the first two modes of vibration. The parameters are used to compute the response of several single and multi-degree of freedom structures with TMDs to different earthquake excitations. The results show that the use of the proposed parameters reduces the displacement and acceleration responses significantly. The method can also be used for vibration control of tall buildings using the so-called mega-substructure configuration, where substructures serve as vibration absorbers for the main structure.

In the present analysis the theoretical natural frequencies and mode shape of the simply supported beam type main vibrating system e.gused in many areas like bridges and double-beam structures is determined. Dynamic vibration absorber is designed and developed at one of the resonant frequencies of the main vibrating system. Natural frequencies and mode shapes of main vibrating system obtained analytically and also by using FEA analysis. The system considered is essentially a modification of the conventional damped vibration absorber and consists of adding, in parallel, a subsidiary un-damped absorber mass in addition to the damped absorber mass. The optimized parameters, to reduce the vibration amplitude have been obtained using MATLAB. Further Finite Element Analysis of the simply supported beam has been done to test the performance of the damped and un-damped vibration absorber for the simply supported beam type main system, using ANSYS. The analysis clearly shows that it is possible to obtain an un-damped anti-resonance in a dynamic absorber system which exhibits a well-dampedresonance.While the bandwidth of frequencies between the damped peaks is not significantly increased, the amplitudes of the main mass are considerably smaller within the operational range of the absorber. A total of four models (model A. model B, model C and model D) are taken for consideration for the damped vibration absorber. By comparing in between those models, it was observed that model (C) gives better vibration suppression and also required less damping ratio for anti- resonance.

THEORETICAL ANALYSIS

The general differential equation governing transverse vibration of a Euler-bernouli beam is given as

$$EI\frac{\partial^4 x}{\partial x^4}(x,t)\ +\rho\ \frac{\partial^2 x}{\partial t^2}(x,t)\ = 0 \qquad (1)$$

Where,
E = Young's modulus of elasticity of the beam,
I = M I of the beam,
ρ= density of the beam,
x = distance from one of the ends of beam,
X = amplitude of vibration,
A = c/s area of the beam,
t = time independent variable.
The solution of Eq. (1) can be expressed as

$$X(x) = C_1\cos\beta x + C_2\sin\beta x\ + C_3\cosh\beta\ \ + C_4\sinh\beta \qquad (2)$$

This equation was solved for transverse vibration ofthe beam and natural frequencies of transverse vibration of the beam are given as,

$$\omega = (\beta)^2 \sqrt{\frac{E}{\rho \ l^4}} \qquad (3)$$

where,

ω = natural frequency in Hz,β = constantchanges with B.C. modes,l = length of beam. Assuming the values as follows:

Density ρ = 7850kgf/m^3,

M.I. of the beam = I = bd^3 / 12=3.922 x 10^{-9}m^4,

(l) = 415mm, (b) = 50mm, (d) = 9.80mm,

Material of the beam= mild steel,

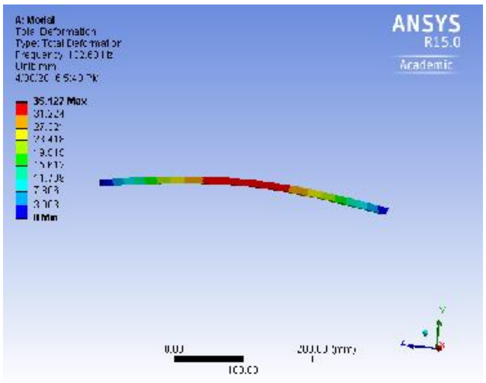
Young's Modulus E = 200Gpa,

ρ =3.8465kg/m,$\beta_n l = n$.

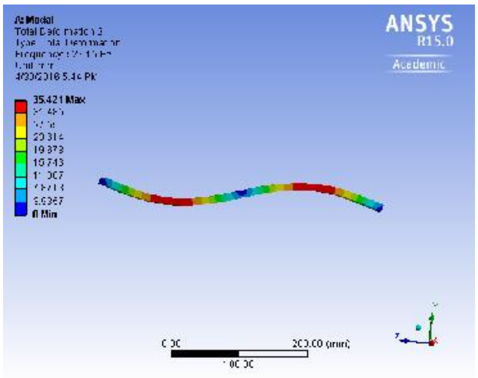
Based on the above theoretical analysis the natural frequencies of the simply supported beam in all the four modes, have been obtained and the subsequently the frequencies have also been obtained by FEA. Table 1 shows comparative values of the natural frequencies in all the four modes. The four modes of the vibrations are as shown in the Figure 1 (a-d)

Table 1: First four natural frequency for different modes of vibrating simply supported beam

Mode	Theoretical frequency(ω_n) in (rad/sec)	FE analysis frequency in (rad/sec)
1	818.35	132.63 x 2π =833.33
2	3273.4	527.15 x 2π =3312.18
3	7365.15	1170.1 x 2π =7352
4	13903.6	1995.2 x 2π =12536.211



(a)



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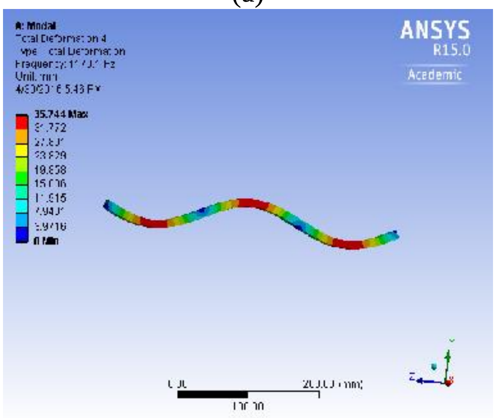

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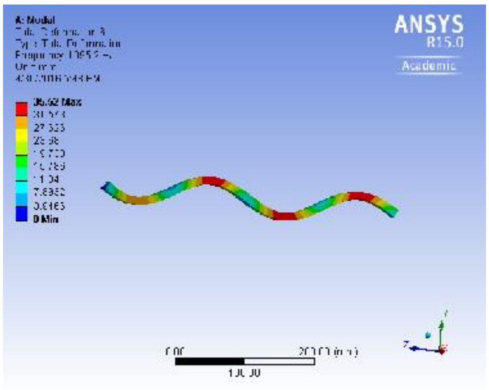

(d)

Figure 1. First 4 Natural Frequencies of vibration of simply supported beam for (a) Mode I, (b) Mode II, (c) Mode III, (d) Mode IV

DESIGN OF DUAL MASS DYNAMIC VIBRATION ABSORBER:

The dynamic vibration absorbers have been designed for simply supported beam type main vibrating system, when main systems are excited at its fundamental naturalfrequency. An absorber mass of 25% of the mass of main system i.e. $\mu = 0.25$ is taken fordesign. Absorber consists of a cantilever beam type arm having specifications as follows:

Rod diameter = 5mm

Length=70 mm

Material =mild steel

Young's Modulus, E=200Gpa,

Density, $\rho = 7850\ kg/m^3$

Moment of Inertia of the beam, $I = \pi d^4/64 = 3.068 \times 10^{-11} m^4$

As absorber arm is made up of 2 rods of 70mm.

MI of the arm, $I = 2I_1 = 3.068 \times 10^{-11} \times 2 = 6.135923 \times 10^{-11}\ m^4$

From mathematical model of absorber system,

mass of absorbers= m_a+m_b=0.25 X mass or the simply supported beam type main vibrating system= 0.25 x 1.5963 =0.3991 kg.

therefore, $m_a = m_b = 0.1995$ kg.

At tunning condition $\omega_n = \omega_a$.

and, $\omega_n^2 = k_2/m_2$, therefore

At fundamental natural frequency, designed $k_2 = 818.35^2 \times 0.1995 = 133.604$KN/m.But secondary mass attached to primary system acts as a cantilever beam. Therefore length of absorber system will be calculated using the formula ,$k_1 = p/y = 3EI/l_l^3$.

so l_1=65.073mm.

DAMPED DYNAMIC VIBRATION ABSORBERS:

Optimum Parameters of Model A:

The amplitude of the machine can be reduced by adding a damped vibration (model A) as shown in Figure 2.

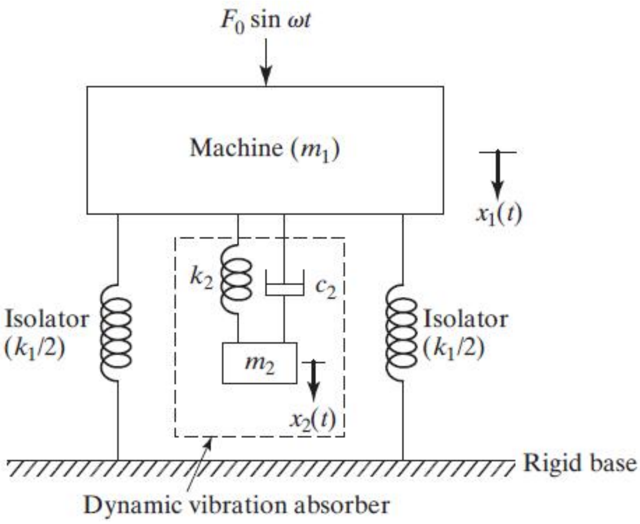

Figure 2. Damped dynamic vibration absorber model A

The equations to obtain the vibration amplitude can be taken from Eq. (4) and Eq.(5), and Eq.(6) indicates the necessary criteria for the absorber to be tunned one.

$$\frac{X_1}{X_s} = \left[\frac{(2\zeta g)^2 + (g^2 - f^2)^2}{(2\zeta g)^2(g^2 - 1 + \mu g^2)^2 + \{\mu f^2 g^2 - (g^2 - 1)(g^2 - f^2)\}^2}\right]^{1/2} \quad (4)$$

And

$$\frac{X_2}{X_s} = \left[\frac{(2\zeta g)^2 + f^4}{(2\zeta g)^2(g^2 - 1 + \mu g^2)^2 + \{\mu f^2 g^2 - (g^2 - 1)(g^2 - f^2)\}^2}\right]^{1/2} \quad (5)$$

$$f = \frac{1}{1+\mu} \quad (6)$$

An absorber satisfying Eq. (6) can be correctly called the *tuned vibration absorber*. Although Eq. (6) indicates how to tune an absorber, it does not indicate the optimalvalue of the damping ratio ζ and the corresponding value ofX_1/X_s . According to Kelly (2000), if the damping ratio is defined as $\zeta = C_2/2m_2\omega_a$, then optimum value of damping ratio can be expressed as:

$$\zeta_o = \sqrt{\frac{3\mu}{8(1+\mu)}} \quad (7)$$

Subsequently, Brock (1946) took a different approach which is quite clever, yet straight forward. No differentiation was needed. Based on the results, he suggested that the optimum damping ratio for constant tuning. The constant tuning is defined as the case when $f = 1$ can be given by

$$\zeta_o = \sqrt{\frac{\mu(3+\mu)\left[1+\sqrt{\mu/(2+\mu)}\right]}{8(1+\mu)}} \quad (8)$$

Optimum parameters of model B:

The dynamic vibration absorber of Model B is as shown in Figure 3.

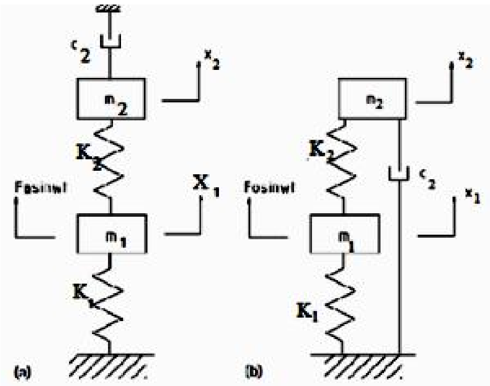

Figure Error! No text of specified style in document.. **Dynamic vibration absorber model B(a)skyhook damper(b)groundhook damper**

The governing equation is as follows:

$$\frac{X_1}{X_s} = \left[\frac{(2\zeta g)^2 + (g^2 - f^2)^2}{(1 + \mu f^2 - g^2)^2(2\zeta g)^2 + [(1-g^2)(f^2 - g^2) - \mu f^2 g^2]^2}\right]^{1/2} \quad (9)$$

$$\frac{X_2}{X_s} = \left[\frac{(2\zeta g)^2 + f^4}{(1 + \mu f^2 - g^2)^2(2\zeta g)^2 + [(1-g^2)(f^2 - g^2) - \mu f^2 g^2]^2}\right]^{1/2} \quad (10)$$

The optimum parameter of model B can be expressed as.

$$f = \frac{1}{\sqrt{1-\mu}} \quad (11)$$

$$\left(\frac{X_1}{X_s}\right)_{o1} = \left(\frac{X_1}{X_s}\right)_m = \frac{2(1-\mu)}{\sqrt{2\mu}} \quad (12)$$

$$\zeta_o = \frac{1}{2}\sqrt{\frac{3\mu}{(1-\mu)(2-\mu)}} \quad (13)$$

Brock(1946) also found the result for constant tuning. In the case of model B, the ordinate of point B is greater than that of point A: We found that the optimum damping ratio is of the form:

$$\zeta_o = \frac{\sqrt{\mu\left[\mu+6-\sqrt{\mu(\mu+2)}\right]}}{4} \quad (14)$$

Brock (1946) employed a perturbation method instead of differentiating a high-order equation. For model B to be optimum, a larger damping ratio is required.

Optimum parameters of model C:

The arrangement of model C type dynamic vibration absorber is as shown in the Figure 4

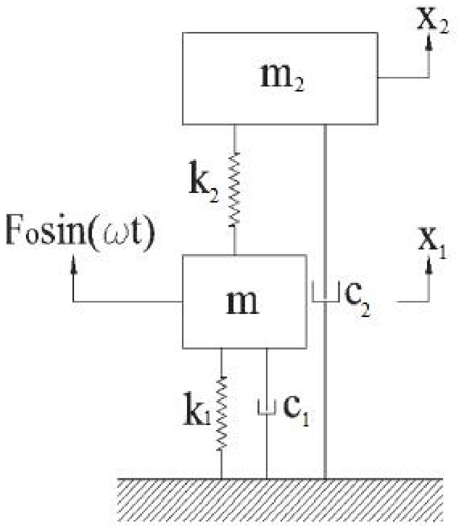

Figure 4. Dynamic vibration absorber model C

The equation of motion for Model C is given by:

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}\begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix} + \begin{bmatrix} c_1 & 0 \\ 0 & c_2 \end{bmatrix}\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} + \begin{bmatrix} k_1+k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} F_0 \\ 0 \end{bmatrix}\sin(\omega\ ) \quad (15)$$

After solving the above equation of motion of model C.The normalized amplitude of the steady-state response of the primary mass is given as:

$$\left(\frac{X_1}{X_s}\right)_{o1} = \sqrt{\frac{\left(1-\frac{g^2}{f^2}\right)^2 + 4\left(\zeta_a\frac{g}{f}\right)^2}{\left(\frac{g^4}{f^2}-\left(\frac{4\zeta_a\zeta_p}{f}+\frac{1}{f^2}+(\mu+1)\right)g^2+1\right)^2 + 4\left(g\left(\zeta_p+\frac{\zeta_a}{f}\right)-\frac{g^3}{f}\left(\zeta_a+\frac{\zeta_p}{f}\right)+g\,\zeta_a\mu\right)^2}} \quad (16)$$

$$\left(\frac{X_2}{X_s}\right)_{o1} = \sqrt{\frac{1+4\left(\zeta_a\frac{g}{f}\right)^2}{\left(\frac{g^4}{f^2}-\left(\frac{4\zeta_a\zeta_p}{f}+\frac{1}{f^2}+(\mu+1)\right)g^2+1\right)^2 + 4\left(g\left(\zeta_p+\frac{\zeta_a}{f}\right)-\frac{g^3}{f}\left(\zeta_a+\frac{\zeta_p}{f}\right)+g\,\zeta_a\mu\right)^2}} \quad (17)$$

Where

$$\zeta_p = \frac{C_1}{2m_1\omega_p}\ , \quad \zeta_a = \frac{C_2}{2m_2\omega_a}$$

Liu and Coppola (2010)have provided the optimum parameter on model C as:

$$f = \sqrt{\frac{1-4\zeta_p^2}{1-\mu}}\ , \zeta^*_{a(o1}\quad) = \frac{1}{2}\sqrt{\frac{3\mu}{2-\mu}} \tag{18}$$

It has been found that with an increase of the damping ratio ζ_p or the mass ratio μ, the optimum tuning parameter *f* decreases and the optimum damping ratio ζ_a increases.It has been found that with an increase of the damping ratio ζ_p or the mass ratio μ , the optimum tuning parameter *f* decreases and the optimum damping ratio ζ_a increases.

Optimum parameters of model D:

The arrangement for this type of vibration absorber is as shown in the Figure 5.

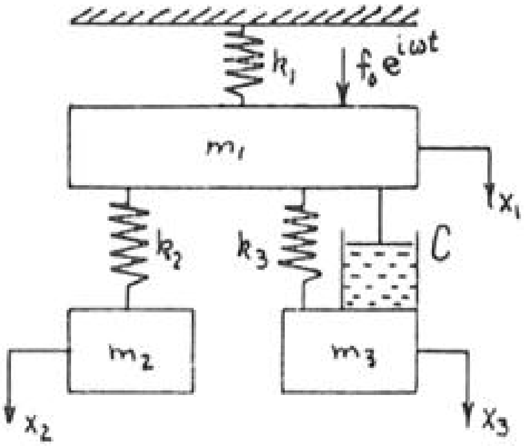

Figure 5. parallel dual mass damped dynamic vibration absorber

The ratio of amplitudes may be expressed as:

$$\frac{X_1}{X_s} = \frac{\left(1-\frac{g^2}{f^2}-\frac{g^2}{h^2}+\frac{g^4}{f^2h^2}\right)+i2\zeta\frac{g}{f^2}\left(1-\frac{g^2}{h^2}\right)}{\left[\begin{array}{l}\left\{(1+\mu_2h^2+\mu_3f^2-g^2)\left(1-\frac{g^2}{f^2}-\frac{g^2}{h^2}+\frac{g^4}{f^2h^2}\right)+\mu_2h^2\left(\frac{g^2}{f^2}-1\right)+\mu_3f^2\left(\frac{g^2}{h^2}-1\right)\right\}\\ +i2\zeta g\mu_3\left\{\left(1-\frac{g^2}{f^2}-\frac{g^2}{h^2}+\frac{g^4}{f^2h^2}\right)+\left(1-\frac{g^2}{h^2}\right)\left(\frac{\mu_2h^2}{\mu_3f^2}+\frac{1}{\mu_3}\frac{1}{f^2}-\frac{1}{\mu_3}\frac{g^2}{f^2}-1\right)-\frac{\mu_2h^2}{\mu_3f^2}\right\}\end{array}\right]} \tag{19}$$

$$D\frac{X_2}{X_s} = \left(1-\frac{g^2}{f^2}\right)+i2\zeta\frac{g}{f^2}$$

$$D\frac{X_3}{X_s} = \left(1-\frac{g^2}{h^2}\right)\left\{1+i2\zeta\frac{g}{f^2}\right\} \tag{20}$$

Where,$f = \omega_3 / \omega_1$, $g = \omega / \omega_1$, $h = \omega_2 / \omega_1$, $\mu_2 = m_2/m_1$, $\mu_3 = m_3/m_1$, $\zeta = C/C_c = {}^{C}/_{2m_2\omega_p}$

$$f^2 = \frac{1-\mu}{(1+2\mu)^2} \tag{21}$$

The required tuning, the so-called favorable tuning, which gives equal amplitudes can be calculated from Equation (21). 36.6 percent of the main mass is required for the absorber mass in order to attain a maximum bandwidth of (g_1- g_2).However, such a mass ratio is too high and prohibitive to tie of any practical use. The parallel absorber appears to be superior todamping ratio such as ζ = 0.32.

RESULTS AND DISCUSSIONS

The effect of damped DVA for different values of ζ and μ in FRF has been analyzed using MATLAB.

Optimisation of model A damped dynamic vibration absorber:

The variations of vibration amplitudes of the main and auxiliary masses of vibration absorber model A,Eqs. (4) and (5), as functions of the frequency ratio (g), using MATLAB are shown in Figure 6. Response and sub-plot response of combined system, mass ratio for model A are shown in Figure 6 (a) and Figure 6 (b) respectively. From the above curve, it can be seen that the non-linear variation of amplitude of vibrating system. For model A, the ordinate of point A is greater than that of point B: The optimum damping ratio is

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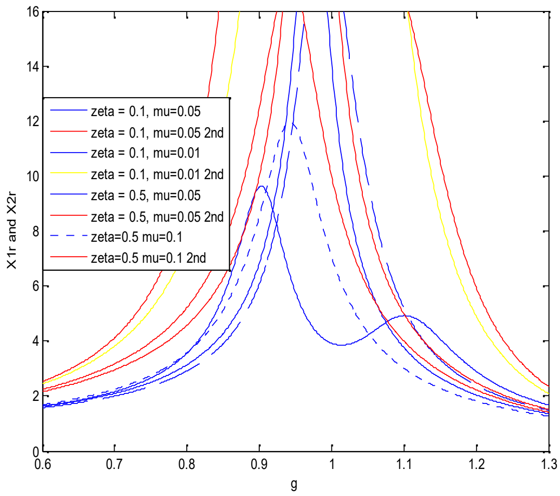
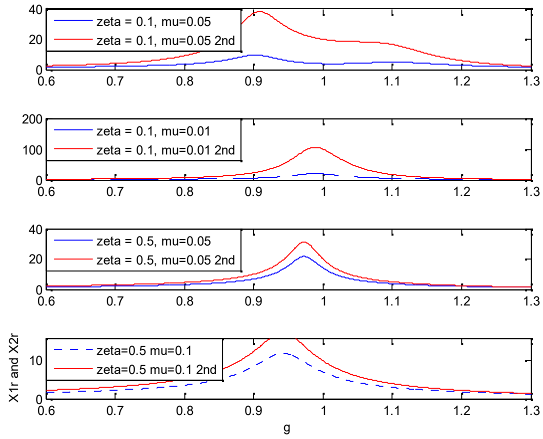
considered to be the value for which the FRF curve passes horizontally through point A and it goings to optimize at $\zeta = 0.27$ and $f = 0.8$ for $\mu = 0.25$.





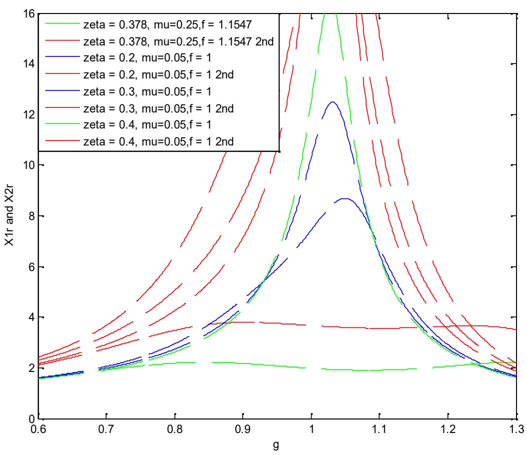
(b)

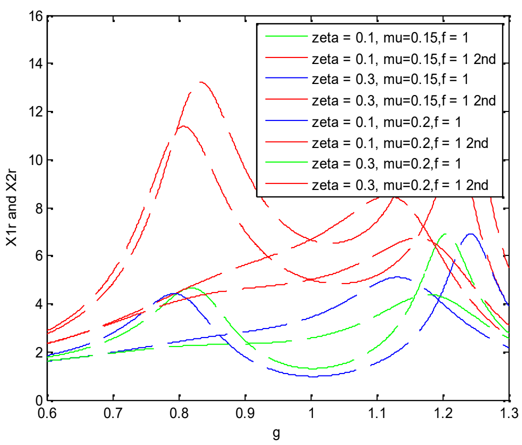
(a)

Figure 6 (a) Response and (b) sub-plot response of combined system using dynamic vibration absorber model A.

Optimisation of model B damped dynamic vibration absorber:

The variations of vibration amplitudes of the main and auxiliary masses of vibration absorber model B, Eqs. (9) and (10), as functions of the frequency ratio (g), using MATLAB are shown in Figure 7.





(b)

(a)

Figure 7Response of combined system, mass ratio for model B with (a) optimal tuning (b) for at constant tuning

From the above curve, it can be seen that the non-linear variation of amplitude of vibrating system. For model B, the ordinate of point B is greater than that of point A. The optimumthrough point B and it goings to optimize at $\zeta = 0.378$ and $f = 1.1547$ for $\mu = 0.25$.

Optimisation of model C damped dynamic vibration absorber:

The variations of vibration amplitudes of the main and auxiliary masses of vibration absorber model C, Eqs. (16) and (17), as functions of the frequency ratio (g), using MATLAB are shown in Figure 8.

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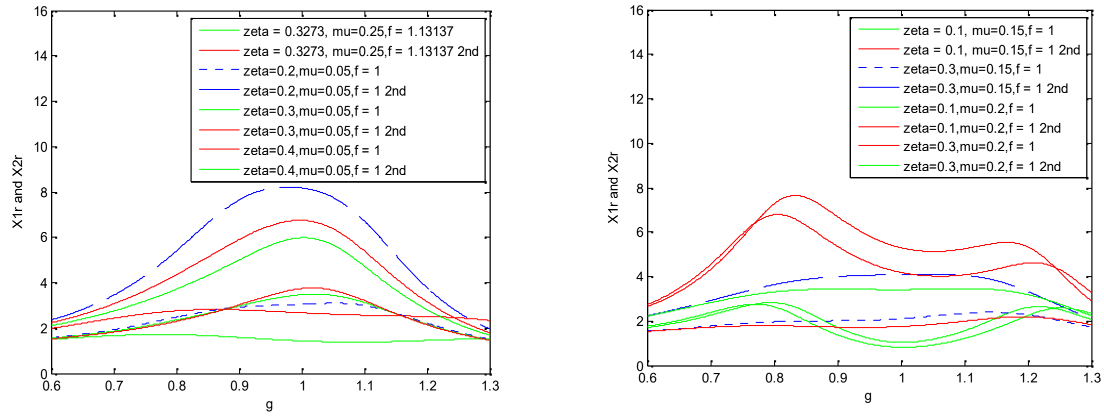


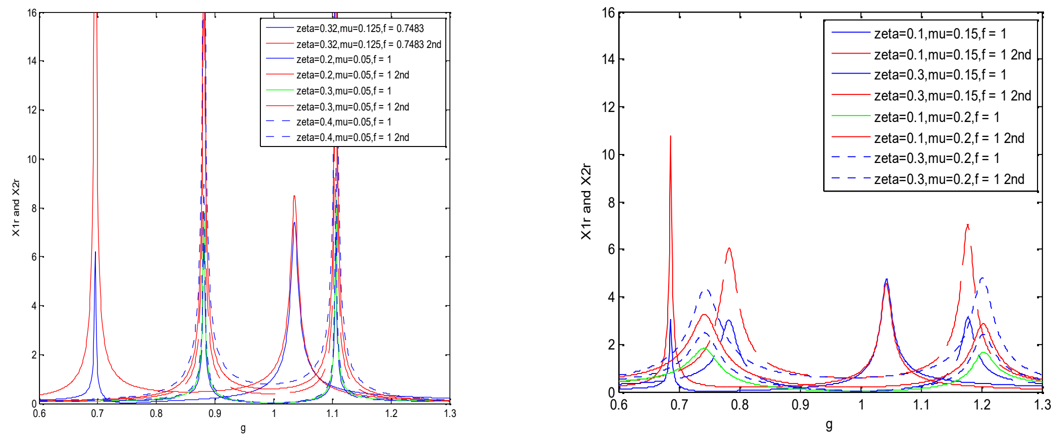
(a) (b)

Figure 8Response of combined system, mass ratio for model C with (a) optimal tuning (b) for at constant tuning

From the above curve, it can be seen that the non-linear variation of amplitude of vibrating system For model C, the ordinate of point A is greater than that of point B: The optimum damping ratio is considered to be the value for which the FRF curve passes horizontally through point A and it goings to optimize at $\zeta = 0.3273$ and $f = 1.13137$ for $\mu = 0.25$.

Optimisation of model D damped dynamic vibration absorber:

The variations of vibration amplitudes of the main and auxiliary masses of vibration absorber model D, Eqs. (19) and (20), as functions of the frequency ratio (g), using MATLAB are shown in Figure 9.



(a) (b)

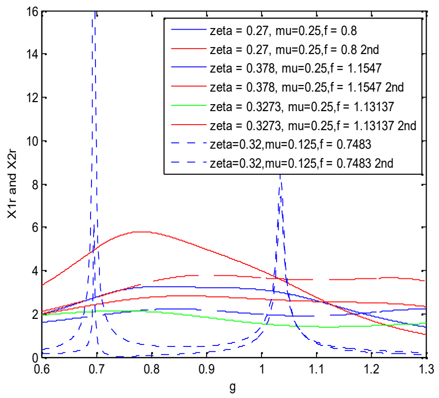
Figure 9Response of combined system, mass ratio for model D with (a) optimal tuning (b) for at constant tuning

As compared to other model, this model suppress the amplitude of vibration so high but at a small range of excitation it shows large amplitude, in order to avoid it we got the optimize damping coefficients of ζ_a=0.32.

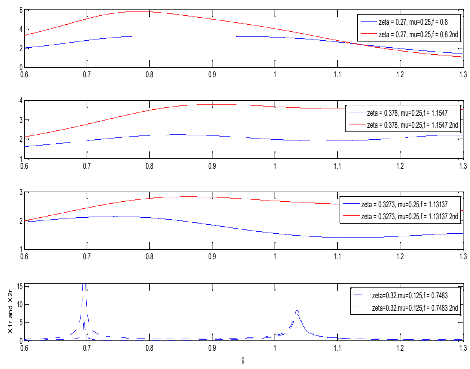
A comparison of all the optimal parameters of all the four models are shown in Table 2 and a comparative response and sub-plot response of the all the four models are shown in Figure 10.

Table 2. **Comparison of the four models at μ = 0.25.**

MODEL	f_{ti}	ζ_o	$\left(\frac{X_1}{X_s}\right)_{f_c\ ,f_t}$	$\left(\frac{X_1}{X_s}\right)_{\zeta_o\ ,f_t}$
A	0.8	0.27	6.1308	3.2581
B	1.1547	0.378	4.2249	2.2228
C	1.13137	0.3273	2.0743	2.1334
D	0.7483(μ = 0.125)	0.32	7.1034	7.3945



(a)



(b)

Figure 10. (a) Response and (b) Subplot response for a comparison of all the optimal four models

CONCLUSION

From Figure 10, it can be seen that the amplitude of the absorber mass is always much greater than that of the main mass. Thus the design should be able to accommodate the large amplitudes of the absorber mass. For the model A, as shown in Figure 10, it is seen that as the mass ratio increases amplitude of vibration decreases. As mass ratio increases the optimum damping also increases. One observation we can made from the Frequency response function's curve is the response curve becomes flatter as the mass ratio increases.

The comparison curve of all 4 model superimposed in one plot as shown in Figure 10shows that, for model B to be optimum, a larger damping is required as compared to model A. Overall, model C gives better vibration suppression and also required damping in between model A and model B. The parallel absorber appears to be superior to the conventional clamped absorber if a comparison is made between the response curves for a damping ratio such as ζ_a = 0.32. The conventional absorber has, for this ratio of ζ_a prohibitively large amplitudes with in the operational range of the vibration absorber. Comparing all the model we conclude that design of primary system should be based on model C at ζ = 0.3273 and f = 1.13137 for μ = 0.25.

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Appendix A: Notations

E = Young's modulus of elasticity of the beam

I = M I of the beam

ρ= density of the beam

x= distance from one of the ends of beam

X = amplitude of vibration

A = c/s area of the beam

t = time independent variable

ω_n = natural frequency of the system

k = stiffness of the spring

$X_{St} = F_0/k_1$ = Frequency deflection of first mass

$\omega_1 = k_1/m_1$ = natural frequency of main system alone

$\omega_2 = k_2/m_2$ = natural frequency of the absorber system alone

$\mu = m_2/m_1$ = ratio of absorber mass to the main mass

$X_s = F_0/k_1$ = Static deflection of the system

$\omega_a^2 = k_2/m_2$ = Square of natural frequency of the absorber

$\omega_n^2 = k_1/m_1$ = Square of natural frequency of main mass

$f = \omega_a/\omega_n$ = Ratio of natural frequencies

$g = \omega/\omega_n$ = Forced frequency ratio

C = damping constant

$C_C = 2m_2\omega_n$ = Critical damping constant

$\zeta = C_2/C_C$ = Damping ratio

f_f = favorable tuning frequency/natural frequency of main mass

f_0 = amplitude of the forcing function

h = natural frequency of undampedabsorber/natural frequency ofmain mass

$i = \sqrt{-1}$

k_1= Spring constant of main spring

k_2= Spring constant of undamped absorber

k_3= Spring constant of damped absorber

m_1 = Magnitude of main system

FRF = Frequency response function