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Decay of Localized Pulse Waves through Viscoelastic Tube of an Arterial System

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ABSTRACT. *In this study,the mathematical modelling of blood flow through the nonlinear pulse wavesin the arterial system.We consider the one-dimensional wave propagation in an infinitely long, straight, cylindrical and inhomogeneous nonlinear viscoelastic tube filled with an incompressible, inviscid fluid. Such a combination of a solid and fluid is considered to be a Kelvin-Voigt model for blood flow in the artery. Using the mass conservation and the momentum theorem of the fluid and radial motion of the arterial tube wall, a set of nonlinear partial differential equations governing the propagation of nonlinear pressure wave equation is obtained. We invoke the reductive perturbation method to the nonlinear equations of the tube and fluid as well as the knowledge of, and then selecting the exponent ~ of the perturbation parameter in Gardner-Morikawa transformation according to the order of the viscous coefficient ¦ we obtain the nonlinear dynamical equation as Korteweg-de Vries Burgers (KdVB.equation. We employ the Sine-Cosine method for the KdVB equation and obtain the travelling wave solution in an arterial system. Results are presented graphically and discussed both qualitatively and quantitatively from the physiological points of view and its clinical relevance is discussed.*

Keywords: Soliton, Viscoelastic tube, Artery, KdV-Burgers equation.

INTRODUCTION

Mathematical models of the cardiovascular system have a close relationship with the nature of blood motion, the deformability of the arterial wall and to predict dynamical patterns in physiological and pathological conditions. The several researchers have been studied through the propagation of blood pressure pulse waves in distensible blood vessels. Such problems have been investigated, especially, in the point of view, their applications to biological problems involving pulse wave propagation in arterial blood vessels and the majority of the work on blood flow deal with linearized models[1, 2].The study of pressure pulsehas a consequenceof solving complexcardiovasculargeometrical problems with engineering and science. In particular, in interest in nonlinear wave propagation, the establishment of the soliton theory is a great achievement in modern nonlinear sciences and this theory has been applied widely to various fields of science.From the 1980's, with growth of nonlinear science, began the investigation of nonlinear waves, especially the solitary waves, inmany researchers has been studied the motion localized pressure waves in the nonlinearviscoelastic tube filled with an incompressible and inviscous or viscous blood[3-10]. Paquerot and Remoissent also studied the propagation of pressure waves and wall displacement wavesin large artery with variation of both radius and youngs modulus[11].

By using the reductive perturbation technique [12], the propagation of pulse waves is investigated to the human arterial system. The dynamical equations such as Burgers equation, the Korteweg-deVries (KdV).equation and the Korteweg-deVries-Burgers (KdVB).equations are obtained from due to balance between the nonlinearity and dissipation or/and dispersion, respectively. Due to the dependence of the

coefficient characterizing the nonlinearity on the initial deformation, the contour of solution changes into stretch ratios. Moreover, this coefficient vanishes with vanishing initial deformation and the evolution equations degenerate to the linear ones. For this particular deformation, we make a plea forhigher-orderperturbation expansion and obtained the modified evolution equations and results are discussed.In recent years, quite someeffective analyticalmethods for obtaining explicit traveling and solitary wave solutions of nonlinear PDEs have been proposed. A variety of sophisticates methods such as, Backlund and Darboux transform[13], Extended tanh function method[14],inverse scattering method [15], Hyperbolic function method[16], Jacobi elliptic function expansion method[17], and F-expansion method[18] and to find the exact traveling wave solutions by the Sine-Cosine[14] methodof the KDVB equation.

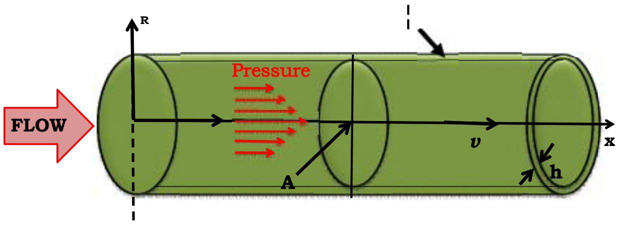

FIGURE 1: Schematic diagram of viscoelastic tube of arteries with propagation of blood pressure pulse waves in*x*- direction.

GOVERNING EQUATIONS AND LONG-WAVE APPROXIMATION

By virtue of the anatomical geometry, the pulse can be represented by a one-dimensional wave. We assume that blood can be regarded as an incompressible and inviscous fluid. Further, our model assumes that arteries are uniform, viscoelastic inhomogeneous cylindrical tubes having nonlinear elasticity in Fig. (1). The laws of hydrodynamics governing the transport of an inviscous and incompressible fluid are the conservation of mass and the momentum equation, given, respectively, by

$$\frac{\partial A}{\partial t}+\frac{\partial (Av)}{\partial x}=0 \tag{1}$$

$$\frac{\partial v}{\partial t}+v\frac{\partial v}{\partial x}+\frac{1}{\rho_0}\frac{\partial p}{\partial x}=0 \tag{2}$$

A third equation describing the radial motion of the wall under the forces exerted by the fluid is necessary in order to have a complete specification of the system [19],

$$\rho_w h\frac{\partial^2 R}{\partial t^2}=P-P_e-\frac{h}{R}\sigma \tag{3}$$

Where ρ_0 and ρ_w are the density of the blood anddensity of arterial wall, respectively.$P_e$the external pressure, σ the extending stress in the tangentialdirection, and *h* the thickness of the tube of radius$R(x,t)$. . We can suppress the unknown pressure $P_e$if we consider an artery already inflated at the diastolic pressure P_0 with radius R_0, thickness h_0. Only considering weak waves

$$A-A_0=\pi R^2-\pi R_0^{\,2}=2\pi R_0\left(R-R_0\right) \tag{4}$$

and defining the small radial elongation of the arterial wall $\varepsilon_0=(R-R_0)/R_0$. In the present work, viscous effect is taken into consideration and the constitutive relation is described by Kelvin-Voigt model

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$$\text{±} \ = K\text{£}_0 + \text{¦}\,\frac{\partial \text{£}_0}{\partial t} \tag{5}$$

where K is the elastic coefficient, η is the viscous coefficient. Assuming that the tube is incompressible, we have

$$Rh = R_0 h_0 \tag{6}$$

Introducing Eqs. (4-6) into Eq. (3). , the equilibrium equation governing the radial motion of the tube is obtained

$$\frac{\text{°}_w h}{2\text{®} R_0}\frac{\partial^2 (A-A_0)}{\partial t^2} = p - \frac{h_0 R_0}{R^2}\left[K\text{£}_0 + \text{¦}\frac{\partial \text{£}_0}{\partial t}\right] \tag{7}$$

Introducing the dimensionless quantities through the definitions $A = \text{®}\, R_0^{\ 2} A'$, $p = p_0 p'$, $t = Tt'$, $x = Lx'$, $A_0 = \text{®}\, R_0^{\ 2}$, $p_0 = \frac{h}{2R_0}$, $L^2 = \frac{\text{°}_w R_0 h}{2\,\text{°}_0}$, $v = (L/T)v'$, $T^2 = \text{°}_w R_0^2$, Wethen have the dimensionless equations from Eqs. (1) , (2) , and (7) ,

$$\frac{\partial A'}{\partial t'} + \frac{\partial (A'v')}{\partial x'} = 0 \tag{8}$$

$$\frac{\partial v'}{\partial t'} + v'\frac{\partial v'}{\partial x'} = -\frac{\partial p'}{\partial x'} \tag{9}$$

$$p' = \frac{\partial^2 A'}{\partial t'^2} + K\left(A'-1\right) + \frac{\text{¦}}{T}\frac{\partial A'}{\partial t'} \tag{10}$$

We adopt the long-wave approximation and solve Eqs. (8-10)by employing the reductive perturbation technique[20-22]. Introduce the following Gardner-Morikawa transformation

$$\text{¶} = \text{£}^{\text{~}}(x' - gt')\,,\ \ \text{²} = \text{£}^{\text{~}+1} gt' \tag{11}$$

where ε is a small parameter measuring the weakness of dispersion and/or non-linearity, and αis a positive constant whose values will be specified later. Assuming that the field parameters can be expanded into the following asymptotic series ofε

$$A' = a_0 + \text{£} A_1 + \text{£}^2 A_2 + ..., \tag{12}$$

$$v' = \text{£} v_1 + \text{£}^2 v_2 + ..., \tag{13}$$

$$p' = p_0 + \text{£} p_1 + \text{£}^2 p_2 + ..., \tag{14}$$

Substituting Eqs. (12-14.and (11.into Eqs. (8-10)and setting the coefficients of like powers ε equal to zero, we obtain the following sets of differential equations.

O(ε.order equations:

$$a_0 v_{1\text{¶}} - gA_{1\text{¶}} = 0\,,\ \ p_{1\text{¶}} - gv_{1\text{¶}} = 0\,,\ \ KA_1 = p_1 \tag{15}$$

O(ε^2.order equations:

$$gA_{1\text{²}} - gA_{2\text{¶}} + v_1 A_{1\text{¶}} + a_0 v_{2\text{¶}} + A_1 v_{1\text{¶}} = 0, \tag{16}$$

$$gv_{1\text{²}} + v_1 v_{1\text{¶}} + p_{2\text{¶}} - gv_{2\text{¶}} = 0, \tag{17}$$

$$g^2 \text{£}^{2\text{~}-1} A_{1\text{¶¶}} + KA_2 - \frac{\text{¦}}{T}\text{£}^{\text{~}-1} A_{1\text{¶}} = p_2 \tag{18}$$

Integrating Eq. (15)with respect to ¶, and taking the integral constant to be zero, we have

$$A_1 = \frac{a_0}{g} v_1\,,\ \ p_1 = gv_1\,,\ \ KA_1 = p_1\,,\ \ g^2 = a_0 K\,.$$

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Let

$$v_1 = u(¶,²)\,,\ A_1 = \frac{a_0}{g}u\,,\ p_1 = gu \tag{19}$$

where $u(\xi,²)$is an unknown function to be solved. Introducing Eq. (19)into the set of Eqs. (16-18) , we obtain

$$\frac{g^2}{ao}A_{2¶} - gv_{2¶} - gu_² - 2uu_¶ = 0 \tag{20}$$

$$gu_² + uu_¶ + p_{2¶} - gv_{2¶} = 0 \tag{21}$$

$$a_0 g£^{2~-1}u_{¶¶} + KA_2 - \frac{¦}{T}£^{~-1}u_¶ = p_2 \tag{22}$$

Eliminating P_2, A_2 and V_2 between these equations, we have

$$Mu_² + Nuu_¶ - £^{~-1}u_{¶¶} + L£^{2~-1}u_{¶¶¶} = 0 \tag{23}$$

Where, $M = \frac{2Tg}{a_0¦}$, $L = \frac{gT}{¦}$, $N = \frac{3T}{a_0¦}$, When ~= 1/2 and O(η=1/2)[23], the evolution Eq. (23)reduces to the following Korteweg-de Vries Burgers (KdVB.equation, we get

$$Mu_² + Nuu_¶ - u_{¶¶} + Lu_{¶¶¶} = 0 \tag{24}$$

Now, let us employ Sine-Cosine method to find out the solitary wave solution of the arterial system. To seek the traveling wave solutions given below

$$u(¶,²) = ª \sin (« ¯) \tag{25}$$

$$u(¶,²) = u(¯)\,,\ ¯ = k¶ - c² \tag{26}$$

where ª, « and are parameters to be determined, the wave number k and velocity c. Putting Eqs. (25)and (26)into Eq. (24), we get,

$$-cMu_– + kNuu_– - k^2u_{–} + Lk^3u_{—} = 0 \tag{27}$$

$$-Mkª\cos(«¯) + 1/2\,Nkª^2\cos^2(«¯) - «\,ª\sin(«¯)\cos^{-1}(«¯) +$$
$$Lk^3\left(«^2\ ^2ª\cos(«¯) - «^2ª\ \cos^{-2}(«¯)\right) = 0 \tag{28}$$

Equating the exponents and the coefficients of each pair of the cosine functions we find system of algebraic equations and solving it’s equation we can obtain the value of unknown parameters as below

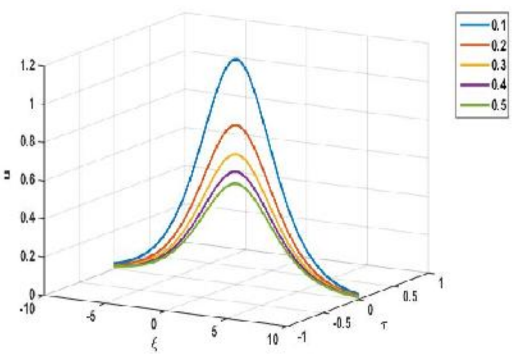

FIGURE 2. The panel displays the soliton propagation through the artery for various values of elasticity (K).

$= -2$, $« = \sqrt{\frac{Mc}{4k^3}}$, $ª = \frac{LMc}{N}$, substituting these parameter values in Eq. (25) we obtain the traveling wave solution

$$u(¶,²) = u(¯)\,,\; ¯ = k¶ - c² \tag{29}$$

$$u(¶,²) = \frac{LMc}{N}\sec h^2\left[\sqrt{\frac{Mc}{4k^3}}(k¶ - c²)\right] \tag{30}$$

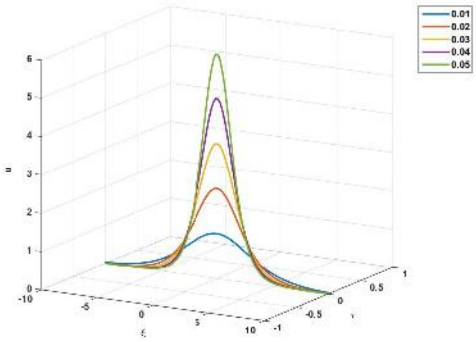

FIGURE 3. The panel displays the soliton propagation through the artery for various values of viscosity (η).

We have plotted Eq. (30) with the parameter valuesare c=0.0017, a_0=0.081, K=0.2, T=6, k=10.1, η=0.01. This represents the dynamical solutions of the viscoelastic tube of an arterial system.We can see the solitary wavepropagation of arterial pressure wave velocityand the wall elastic value K=0.1 to 0.5 increasing the pulse wave amplitude graduallydecayin Fig. (2)and viscosity η=0.01will be increased the soliton amplitude (pressure wave)is increased in shows Fig. (3). The pulse wave variation related to thewall parameter can be attributed to the interaction of the arterial wall with the fluid flow. The arterial elastic limit is increasing to decay in soliton amplitude at minimum velocity if the pulse wave velocity is maximum, while the soliton amplitude is never decay. When the pressure is raised within an elastic vessel, the vessel walls are deformed. An arterial wall viscosity increasing the pulse wave velocity is reduced concurrently wall elasticity is also increased and the maximum systolic diameter of the artery reached during arterial distension is highly dependent upon the level of arterial viscosity.Viscosity of the arterial wall has a little effect on frequency but does play an important role in the spatial pressure distribution.Thus, the effect of a change in tube viscoelasticity on pressure pulse wavepropagation was examined.

CONCLUSIONS:

In the present paper, the propagation of solitary waves in a viscoelastic tube filled with incompressible inviscid fluid is studied. In the long-wave approximation the system of equations governing the propagation of nonlinear pressure wave in a fluid-filled viscoelastic tube is transformed into KdVB equation by using the reductive perturbation method. The travelling wave solution derived from Sine-Cosine method. The soliton solution of KdVB equation can well interpret pulsatile character of arterial blood flow and also explored dissipation and dispersion effect of viscoelastic wave natures. The pressure pulse described by the Burgers equation attenuation of amplitude due to dissipation and its shape is simultaneously distorted due to nonlinearity.As it is well known, the messages carried in the arterial pulse waves are of important applications inclinical diagnosing. Therefore, some results from our study are helpful for discovering diseases of the arteries and inferring the condition at the heart.

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