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Modeling and Simulation of Blood Flow in Tapered Stenosed Arteries in the Presence of Externally Applied Transverse Magnetic Field

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ABSTRACT
The effect of transverse magnetic field applied externally on stenosed tapered artery has been studied to find the behaviour of blood flow. The laminar, incompressible and fully developed flow of blood is studied taking into account the variable viscosity. The blood is considered as Non Newtonian in core region and Newtonian in peripheral region which resembles the real life situation. The constitutive equation of blood is represented by Herschel–Bulkley model in core region. The simulations are carried out for one of the important flow characteristics axial velocity and the behaviour of blood flow is analyzed. Numerical results are reported for different values of the physical parameters of interest. It has been observed that axial velocity is affected in tapered stenosed artery and can be regulated with the help of transverse magnetic field applied externally.

KEYWORDS:Stenosis, Magnetic field, Blood flow, Herschel-Bulkley model, Variable viscosity

1. INTRODUCTION
Atherosclerosis is an arterial disease which leads to malfunctioning of the cardiovascular system [4]. The thickening of an artery is the initial stage in the progression of atherosclerosis which disturbs blood flow, resulting in stenosed artery [1, 4]. When a stenosis is developed in an artery, its serious consequences are the increased resistance and the associated reduction of blood flow [1]. Blood behaves like an electrically conducting fluid which simultaneously exhibits magnetization property under strong magnetic field and the flow is influenced by the presence of magnetic field [1, 3, 6, 8]. The study of blood flow in stenosed arteries has become a key research area as it is more relevant in medical field due to increase in arterial diseases and hence it is important to analyze fluid dynamical aspects of blood flow through a stenosed artery for the fundamental understanding of circulatory disorders.

The main objective of this study is to find the effect of transverse magnetic field applied externally on blood flow in stenosed tapered artery. As blood is shear thinning liquid, viscosity of blood is inversely proportional to shear rate when flowing in core region. So, the variable viscosity of blood which depends on percentage volume of erythrocytes is taken into account in order to improve resemblance to the real situation [5]. The expression for axial velocity is proposed using numerical method and the behaviour in the presence of externally applied transverse magnetic field is studied. The effect of magnetic field, tapering angle and flow

behaviour index has been studied exclusively, through which the link between biomechanics and arterial disease can be comprehended.

2. MATHEMATICAL FORMULATION AND SOLUTION

It is known that when blood flows through arteries, there exists a cell free plasma layer near the wall and in the vicinity of a stenosis, where the shear rate is low and hence blood exhibits non-Newtonian behaviour in core region and Newtonian in peripheral region [1, 2]. Since blood behaves differently in core and peripheral regions, in the present study, blood is modeled as a two fluid model where in core region as Herschel-Bulkley model and in peripheral region as Bingham plastic model [2, 3, 4]. In both the cases low yield stress is assumed as blood requires finite stress before it begins to flow.

The geometry of stenosed tapered artery with tapering angle φ is as shown in the Figure 1. If $\varphi < 0$ then the artery is converging, $\varphi > 0$ then diverging and if $\varphi = 0$ then the artery is non tapered.

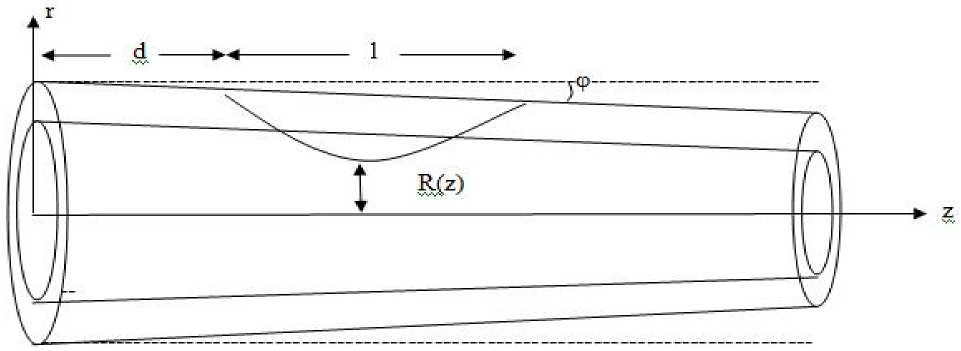

Figure 1. Schematic representation of stenosed tapered artery with tapering angle φ

The non dimensional mathematical model of tapered stenosed artery is given by

$$R(z,t) = \{(R_0 + md) + [m - (h/l)](z-d) + [h/l^2](z-d)^2\}\, a(t) \quad where \quad d \le z \le d+l$$

$$R(z,t) = (mz + R_0)\, a(t) \quad otherwise \tag{1}$$

$$where \quad a(t) = 1 - b(\cos t - 1)e^{-bt}, \quad b = 0.1 \quad for \quad blood. \tag{2}$$

Here R_0 is radius of un-constricted non tapered part of artery, R is the radius of artery in stenosed part, $m = tan\, \varphi$ where φ is the tapering angle and $h = 0.4R_0 \sec \varphi$, height of stenosis.

Considering an axially symmetric, laminar, steady and fully developed flow of blood, an incompressible fluid with externally applied transverse magnetic field and negligible electric field, the non dimensional form of governing equation is [1, 7],

$$\frac{-\partial P}{\partial Z} + \frac{1}{r}\frac{\partial (rT)}{\partial r} + F_1 \frac{\partial H}{\partial Z} = 0 \quad where \quad F_1 = \frac{kMH_0}{\circ u_0^2} \tag{3}$$

where H is magnetic field intensity, M is magnetization, k is magnetic permeability of vacuum, T is shear stress, P is pressure.

The boundary conditions are $u = 0$ at $r = R(z, t)$,

$$\frac{\partial u}{\partial r} = 0 \text{ at } r = 0$$

$$T \text{ is finite at } r = 0 \tag{4}$$

The non dimensional form of constitutive equations of blood in core region is given by

$$\frac{\partial u}{\partial r} = 0 \ if \ T \le T_0 \ where \ \ n > 1 \ or \ n < 1 \qquad (5)$$

$$T = T_0 + [F_2(-\frac{\partial u}{\partial r})]^{\frac{1}{n}} \ if \ T \ge T_0$$

$$Here \ \ F_2 = \frac{\mu}{R_0^{n-1} u_0^{2n-2}} = F_3[1+0.5\{1-r^s\}], \ F_3 = \frac{\mu_0}{R_0^{n-1} u_0^{2n-2}}$$

and $\mu = \mu_0[1+\beta He(1-r^s)]$ which is valid only for dilute suspension of erythrocytes [5]. Here μ_0 = viscosity of plasma, He = maximum hemotocrit at the centre of artery = 0.2, β = 2.5 for blood, T_0 is yield stress, ρ is density, u_0 is velocity of blood, u is axial velocity component, r is radial component, $s \ge 2$ is the parameter determining the exact shape of the velocity profile for blood and n is the flow behaviour index.

The non dimensional form of constitutive equations of blood in peripheral region is given by

$$T = T_0 + F_4\left(-\frac{\partial u}{\partial r}\right) \ if \ T \ge T_0 \ where \ \ F_4 = \mu, \ a \text{ constant.}$$

$$\frac{\partial u}{\partial r} = 0 \ if \ T \le T_0 \qquad (6)$$

Following values are used to determine the constants F_1, F_3 and F_4 and to study axial velocity profile:

μ = 3.5 cP, R_0 = 0.25 cm, u_0 = 30 cm / sec, k = 1, H_0 = 0.2 Tesla, μ_0 = 1.5 cP, M = 2 amp / sec, ρ = 1.055 gm/cm^3, d = 1 cm, l = 4 cm, s = 2.

Solving (1), (3), (5) and (6) using (4) we get,

$$u_c = \int_r^{R(z,t)} \frac{\left[\frac{r}{2}\left(\frac{\partial P}{\partial Z} - F_1 \frac{dH}{dZ}\right) - T_0\right]^n}{F_3\left[1+0.5\left(1-r^2\right)\right]} dr \ \ where \ \ r \ ranges \ \ from \ \ 0 \ to \ R_c \qquad (7)$$

$$u_p = \int_r^{R(z,t)} \frac{\left[\frac{r}{2}\left(\frac{\partial P}{\partial Z} - F_1 \frac{dH}{dZ}\right) - T_0\right]}{F_4} dr \ \ where \ \ r \ ranges \ \ from \ \ R_c \ to \ R \qquad (8)$$

Here u_c and u_p are axial velocity components in core region and peripheral region respectively.

3. RESULTS AND DISCUSSION

Matlab 7.10.0 is used to compute dimensionless u_c and u_p for various values of non dimensional fluid parameters and the results are presented graphically in Figure 2 to Figure 5. The effect of magnetic field is visualized in all the graphs to control blood flow.

It is observed from Figure 2 and Figure 3 that the axial velocity decreases as r increases in core region irrespective of tapering angle, but axial velocity decreases drastically in converging artery ($\varphi < 0$) when compared to the decrease of axial velocity in diverging artery ($\varphi > 0$). Also, it is clear that the application of magnetic field slows down the variation in axial velocity, through which the blood flow can be regulated. This is due to Lorentz's force induced by applying magnetic field which opposes the flow of blood.

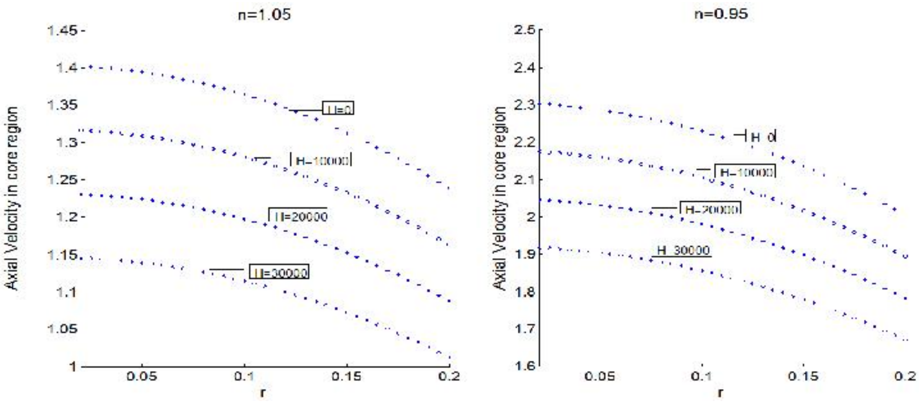

Figure 2. Velocity profile in core region w.r.t. *dH/dZ* at *z* =2 when *φ* = 0.05, *T₀*=0.02, *dP/dZ*=50

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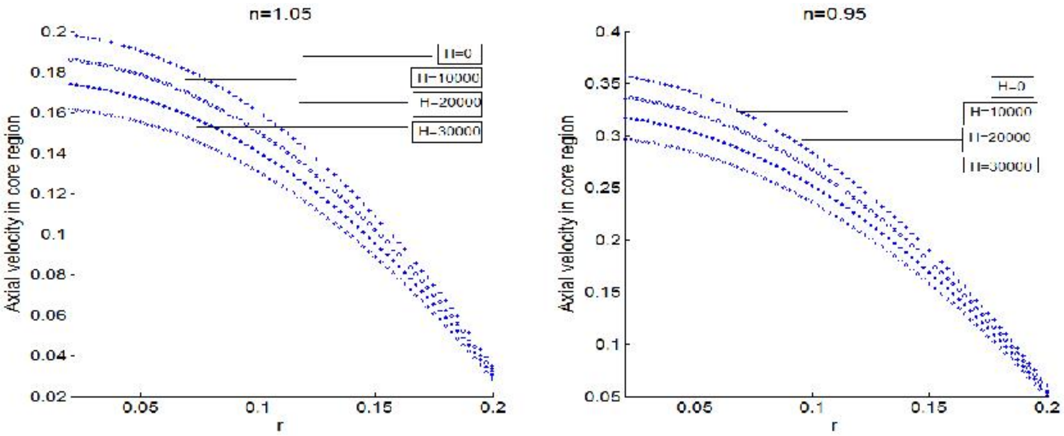

Figure 3. Velocity profile in core region w.r.t. *dH/dZ* at z = 2 when φ = - 0.05, T_0=0.02, *dP/dZ* = 50

Fig. 4 illustrates that the variation in pressure gradient plays an important role in slowing down blood flow. Balancing pressure gradient and magnetic field gradient plays an important role in reducing the speed of blood. The above figures also depict the behaviour of axial velocity when blood is considered as shear thinning ($n < 1$) and shear thickening ($n > 1$).

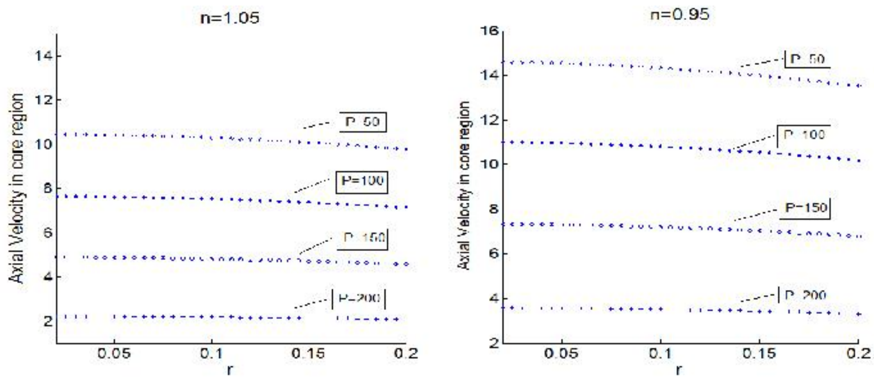

Figure 4. Velocity profile in core region w.r.t. *dP/dZ* at *z* = 4 when φ = 0.05, T_0 = 0.02, *dH/dZ* =20000

Fig. 5 exhibits that there is a linear relationship between radial coordinate and axial velocity in peripheral region. It can also be seen that the type of artery (converging, non tapered, diverging) has a major role in deciding the behaviour of axial velocity. Magnetic field plays an important role here also in reducing the velocity of blood.

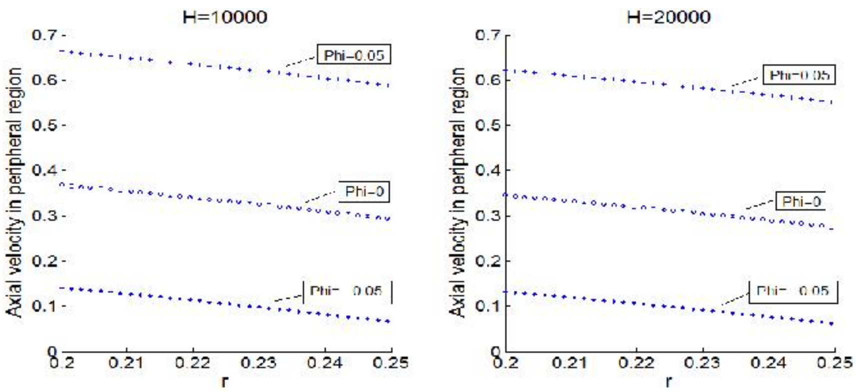

Figure 5. Velocity profile in peripheral region w.r.t. φ at *z* = 2 when *dP/dZ = 50*, T_0=0.02

4. CONCLUSION

In this work, the effect of externally applied transverse magnetic field in stenosed tapered artery is studied by considering variable viscosity when blood is modelled as Herschel-Bulkley model. Axial velocity is one of the important flow characteristics which play an important role in the study of blood flow. Yield stress, pressure gradient, magnetic field gradient, flow behaviour index and tapering angle are the key parameters which affects axial velocity. It is also observed that the behaviour of axial velocity with variable viscosity is in good agreement with the published results.

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